ON THE „RESIDUE" IN HEARING

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Aan mijn Ouders
Aan mijn Vrouw

CONTENTS

INTRODUCTION

0.1. Ever since the time Ohm formulated his law governing the analysing possibilities of the auditory system, the greater part of auditory research has been devoted to the Fourier analysis of complex sounds and the perception of its basic element as a pure tone. The special role of the pure tone in the ear attractively matches the use of Fourier analysis throughout the whole field of acoustical engineering.

The fact that the Fourier components are perceived separately points toward a minimum of interaction in the auditory system. This, as well as the finding that the sound impression is to a certain degree independent of the phases of the components, is easily accounted for when one assumes that each component stimulates a specific channel. In the physiological sense these channels are found in the collection of subsequent points along the basilar membrane (Helmholtz). This one-to-one correspondence is commonly referred to as place principle.

Mathematical treatment of the dynamics of the cochlea (reviewed by Zwislocki, Z-4) as well as experimental determination of the vibration patterns (Békésy, B-4) reveal that the action due to a pure tone is more or less localized. The place theory proper states that the *place* of maximum action is uniquely associated with the psychical dimension *pitch*. The selectivity of the experimental response curves is too low to account for the auditory analysis. Consequently, various mechanisms were proposed capable of improving the mechanical selectivity (Huggins and Licklider, HL-1). Even when the amount of sharpening is limited to a value in accordance with the critical bandwidth (Fletcher, F-5, F-3) the mechanism is still complex and necessitates an accuracy not commonly found in physiological processes.

For the purpose of overcoming this difficulty, the frequency as a succession of events distributed in time is thought to give information about pitch, in addition to the information given in terms of place (Wever, W-1, W-2). The reality of this so-called "frequency principle" in the actual auditory process is still a matter for discussion, though. Experiments on binaural beats (Licklider and Webster, LW-1) and directional hearing (Kietz, K-1) are commonly considered to prove some preservation of phase along the auditory pathways. This evidence is clearly in favour of a frequency principle. A confirmation from experiments on single nerve responses would be desirable, but the results (Tasaki, T-1) are as yet inconclusive in this respect.

0.2. The fact that in normal hearing the tone quality of a complex is perceived as a whole, and an analysis into pure tones is never performed, has been considered only rarely as a primary property of the ear.

When one wants to retain the pure place theory, this may lead one to postulate a peripheral synthetizing mechanism. When a complex tone is presented, the distribution of vibrations in the cochlea does not reveal specifically the relations between harmonics that are so striking when expressed in frequency ratio's. When, in addition to the place, the mechanism acts upon the frequencies as repetition rates it is easy to conceive of a mechanism capable of collecting harmonically related components together. In addition, the special role of the integral numbers in hearing is more directly explained.

0.3. The formant idea (Fletcher, F-3), expressing the fact that a certain specificity of timbre is closely connected with local maxima in the sound spectrum and largely independent of pitch, has been incorporated into auditory theory by Huggins (H-4). From an evolutional point of view, he argues that the analysis of sounds into damped waves, instead of stationary ones, is the most general property of the ear. This analysis, according to Huggins (H-3), is performed by comparing the phases of vibrations at neighbouring points on the basilar membrane. The intervention of synchronous nerve actions is introduced for this purpose only, no specific reference being made to the question whether it is used as a cue for pitch determination or not.

Continuing the evolutional line of thought, one can imagine that fully periodic successions were playing an increasingly important role during development. The temporal information, as defined by Huggins, can be thought to have attained through the ages the special impression of pitch, attributed to the sound as a whole.

0.4. Huggins' theory can be regarded as a combination of place and frequency principles different from the combination implied by Wever (W-2). Moreover, the mechanism assumed to be responsible for the separation and the evaluation of structural and temporal information is more specific than Wever's original concept (Wever only describes the way the signals are transmitted but does not explicitly mention the measuring device).

A theory based more directly on the place and frequency principles has been developed by Licklider. The properties of perception of complex sounds are given a great weight in his evaluation. In order to bring spatial and temporal information into one domain, he uses a scheme analogous to autocorrelation (L-2). The resulting neural activity is especially adapted to computations that are reasonable for neurophysiological processes. The result of this "analogue computing action" appears in such a form that the subjective pitch associated to certain complex sounds is easily explained.

It is clarifying to follow somewhat closer the line of thought leading to experiments on subjective tones and to the construction of tone complexes that produce them.

0.5. A mechanism that is normally stimulated by two widely different kinds of information, such as place and frequency, can probably be deceived by confrontation with a signal that carries a very striking information of one kind but lacks the corresponding one of the other. A pronounced periodicity is generally present in a signal containing a large number of harmonics, and it remains present when the Fourier component of the corresponding frequency, i.e. the fundamental, is missing. When the ear detects the periodicity by way of the frequency principle, one can expect that a pitch corresponding to the lacking component will arise.

The results of experiments on the existence of this phenomenon, the "case of the missing fundamental" (Stevens and Davis, SD-1), have been far from unanimous, however. Positive and negative evidences succeeded one another regularly. An experiment proving that the subjective tone is entirely due to non-linear distortion can be considered a negative evidence. Such a finding is in strong favour of a pure place theory. A positive evidence is obtained when it is shown that the phenomenon is not caused by re-introduction of the fundamental into the sound. When the subjective tone is found to be related solely to the waveform, one is justified in stating that the frequency principle applies.

Further investigation into the case of the missing fundamental seems to be of considerable importance for auditory theory.

0.6. It is possible to describe the phenomenon under study in yet a different way. Almost beyond doubt, the mechanical activity due to a complex of high tones is confined to a relatively narrow region in the basal turn of the cochlea. Thus only the channels of reception for high tones are stimulated. When a low subjective tone arises, which is not caused by distortion, it must be perceived by the high tone channels. Accordingly, it can be expected to exhibit properties different from those of pure tones.

Schouten, who studied this phenomenon extensively in 1940, was able to demonstrate this. He found that the higher part of the spectrum is not resolved into its components by the ear, and that this part gives an impression of low pitch, equal to that of the fundamental of the peaked waveform used. He introduced the name "residue" for this combined impression of high harmonics (S-4). He observed that a residue and the corresponding fundamental could be heard simultaneously without interaction. This absence of interaction was even found when a pure tone and a residue of slightly different pitch were presented. Schouten reported that in such a case no beats were heard.

The view that the residue is connected with the high tone channels only is proven by Licklider (L-2) in a different way. He found that a masking signal consisting of low frequency noise left the residue untouched, although the low frequency channels were saturated.

0.7. Several authors hold the view that this phenomenon is entirely due to distortion. Fletcher (F-2) described the missing-fundamental effect in the years 1924-1934, suggesting that distortion plays the dominant role. Hoogland tried to solve the controversy in 1953 (H-2). He found that the low-pitched subjective component corresponding to a complex of high tones behaved like a distortion product. Consequently, he denied the existence of

a low-pitched residue, and ascribed to Schouten a negligence in testing for the absence of distortion.

It is to be concluded that some critical factor plays a role in the process, as more recent reports (Thurlow and Small, TS-1) again confirm the residue hypothesis, even in the case of chopped noise (Miller and Taylor, MT-1). From a comparison between the various publications it is not possible to identify this factor, because in many instances the ranges of complexes used were rather limited. The present situation calls for a fairly elaborate investigation of the effect. This is the subject treated in this thesis. During the course of the research stress was laid upon the properties of the residue, rather than on the explanation of the divergence of views. The results of the research imply in fact quite a restriction of the number of possible theories. Regarding the mechanism underlying the phenomenon one cannot be positive, although various possibilities can be listed in order of likelihood.

In the first chapter the results and methods of previous investigations are summarized. Some classification of the methods is then necessary in order to investigate whether there is one single way of attacking the problem that can provide useful comparison between various methods (Chapter II). The actual experiments, performed by use of the modulation method, are roughly divided into qualitative (Chapter III) and quantitative ones. The description of the latter kind of experiments is preceded by a short discussion dealing with the division into frequency and phase effects (Chapter IV). The quantitative experiments are treated in Chapters V and VI, the latter chapter covering experiments that are related to certain aspects borne out by theoretical considerations. Chapter VII contains a discussion of results obtained by other experimenters. The final, theoretical chapter deals with suggestions as to various possibilities for a mechanism capable of explaining the results. In the Appendix the equipment used in the experiments is described.

Chapter I

THE RESIDUE

1.1. The pitch of musical tones is generally found not to be affected by severe suppression of the lower frequencies. This fact was investigated by Fletcher in the years 1924 to 1934 (F-1, F-2, F-4). His experiments were performed with a ten-tone generator. Components of frequencies 100, 200, etc. to 1000 c/s inclusive were produced either by coupled generators (F-4, FM-1) or by independent generators (F-2). When a few adjacent components were sounded together a pitch impression corresponding to the fundamental arose when the sound level surpassed a critical value.

Since this kind of behaviour is typical for a distortion product, Fletcher considered the effect to be due to non-linear distortion at a stage prior to the selective distribution of different frequencies to different places. Quantitative calculations of the loudness of the common difference tone re-introduced by non-linearity (Fletcher, F-3, F-2), established the distortion theory for the high levels used. Consequently, the pitch was stated to be determined by either the fundamental or the difference tone, depending upon which one is louder. For a complex consisting of odd harmonics of 100 c/s a tendency for separation of pitch from that of the difference tone was reported. Such a signal was considered devoid of all tonality, yet the difference tone (here 200 c/s) was audible.

In his Physical Review paper Fletcher states: "When five or more consecutive components were used, the pitch seemed to remain the same for low values of the loudness even down to zero, although for these very low values it was very difficult to judge pitch" (F-1). This feature is not in agreement with the general behaviour of a distortion product. The explanation in terms of non-linear distortion was not tested in a direct way.

1.2. In order to investigate the analysing possibilities of the ear, Schouten used an optical siren for producing his test sounds (S-2).

One period of the desired waveform, plotted in polar coordinates, was cut out of an opaque sheet of paper. A few of such masks were simultaneously scanned by a rotating plane beam of light. The principal signal consisted of a series of equidistant pulses of frequency 200 c/s.

On listening to the sounds thus produced, a sharp tone with a pitch corresponding to the fundamental is perceived. By suitable manipulations the fundamental component can be cancelled (Fig. 1.2.1). Subjective adjustment of complete suppression coincides with objective adjustment. Therefore, non-linear distortion is not important in this experiment. When cancellation is complete, the remaining sound does retain the original pitch (S-1, 1938). Obviously the remaining harmonics produce together a sound with a pitch equal to that of the fundamental, though of a totally different tone quality.

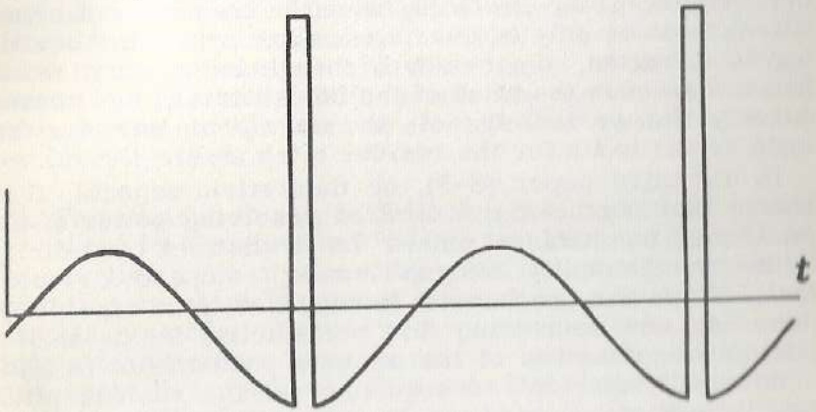


Fig. 1.2.1 Waveform of pulse series when the fundamental is cancelled.

The most prominent subjective component of this sound was a sharp tone produced by the combined impression of the highest harmonics. This component was inaccessible to further subjective resolution and for this reason it was named by Schouten a "residue" (S-4). In the case under investigation the residue had a pitch equal to that of the fundamental of the complex. Experiments are commonly restricted to cases where such a pitch is present, because the most interesting properties may be expected for tonal residues.

An important property of the tonal residue was discovered by Schouten. By exploration with a search tone he ascertained that no trace of fundamental was present in the perceived sound. The result was expressed by stating that a residue and a pure tone of nearly the same pitch do not produce beats.

No direct evidence about the mechanism of pitch perception for the residue being available, the next question is what parameters determine the pitch. Both the frequency difference of the Fourier components and the periodicity of the waveform deserve attention. The influence of the two factors can be separated by use of a so-called "symmetric" signal, i.e. a series of equidistant pulses of alternating sign. Here the separation between the Fourier components is twice the frequency of repetition, because only odd harmonics are present. According to Schouten, the residue, though being very weak, had in this case the pitch of the fundamental, and consequently the periodicity of the waveform was decided upon as the basis for the residue pitch (S-4).

In his third paper (S-5), on theoretical aspects, it is shown that postulating a limited resolving power of the analysing mechanism makes for a distinct periodicity in the mechanically analysed sound (compare Cremer, C-1). When the mechanism is supposed to be capable of detecting and measuring this periodicity, the basis of a possible explanation of the residue phenomenon is laid.

Several important consequences of the residue pitch are immediately apparent. The stability of the pitch of speech sounds under conditions of severe suppression of lower components, such as prevail in telephone and hearing-aid communication, is easily accounted for. The strike note of bells may be due to creation of a residue by the interaction of certain nearly harmonically related tones amongst the partials of the bells (Schouten, S-3).

1.3. A second important property of the residue is described by Schouten in his review paper of 1940 (S-3). By means of a double modulation process all components of a complex were displaced in frequency by a constant amount. This made the original complex inharmonic. It was found that the residue displayed a pitch shift in the corresponding direction. The effect was qual-

itatively demonstrated by using a musical selection. By way of rough comparison it was decided that the relative pitch shift corresponded to the relative frequency shift of the components in the 1000 to 2000 c/s region.

This revealed that the pitch in the musical sense is not determined by the fundamental or the common difference tone, because the pitch shift does not correspond to the frequency shift of the fundamental, neither does it remain constant like the difference tone. The pitch is thus determined by the residue pitch. The qualitative estimate of the pitch shift leads one to the conclusion that musical residues may probably be produced by parts of the spectrum falling in the ear's principal range of frequencies.

It is to be noted that Fletcher performed a similar experiment. He states that a frequency increment of 30 c/s of all components (compare section 1.1) "destroys the musical quality which the original tone possessed" (F-2).

Experiments on inharmonic signals from one of the main items of interest and are chosen as the starting point of the research. Before entering the description of the experiments we want to continue our review of earlier investigations.

1.4. The residue theory of Schouten was severely criticized by Hoogland in 1953 (H-2). The components of his complex were produced by separate tone generators, tuned to a constant frequency difference as accurately as possible. In order to avoid any interaction between complex and subjective tone these two were chosen rather wide apart in frequency. This also facilitated the search for the origin of the latter. Results were, in short, that a subjective tone of pitch equal to that of the fundamental appeared only when the complex surpassed the sound level of 60 db.

All further experiments indicated that it was created as a combined difference tone in the ear. As a second result it was found that it is perceived along the channels normally used for tones of corresponding frequency.

These findings forced Hoogland to deny the existence of the residue pitch and to support Fletcher's view. He considered Schouten's results to be the effect of an undiscovered distortion in the apparatus. According to Hoogland, the difference tone thus created could not be found by the wave analyser because of irregular variations of its frequency, common to all semi-mechanical instruments. These variations of frequency were thought responsible for the reported absence of beats with an external pure tone as well. We will later on (Discussion II) deal with the merits of this explanation.

1.5. For physiological experiments frequent use is made of tone pips, i.e. short tones produced by repetitious excitation of a resonant circuit so that the fundamental is missing (Fig. 1.5.1). In 1951 a paper appeared concerning the subjective impression of such tones when the repetition rate increases into the audible range (Davis and collaborators, DSM-1). It proved rather difficult to compare the pitch of the sound with that of a pure tone. Some listeners, however, made fairly consistent adjustments and matched the pitch of the buzzing sound to a tone of about the repetition frequency or an octave higher.

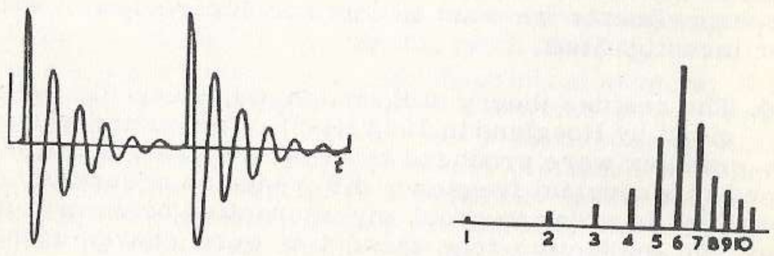


Fig. 1.5.1 Tone pips. Left: waveform. Right: spectrum.

Quite recently experiments along the same line were reported (Thurlow and Small, TS-1). Again it proved possible to ascribe a pitch sensation, corresponding to the repetition rate, to a pulse series fed through a narrow filter that cuts out the lower components. A peculiar effect was noticed when two pulse trains of nearly the same frequency were used simultaneously. The authors report the perception of a "sweep note" the frequency of which seems to be related to the time interval between

the pulses with the smallest separation. When the pulses of one series occurred in about the centre of the time interval of the other series, the octave jump reported by Seebeck (S-7) and Schouten (S-4) was confirmed (see section 3.1).

1.6. Phase and pitch effects of complex sounds are reported by Mathes and Miller (MM-1). Two-component signals were produced by amplitude modulation (the carrier was suppressed in the modulator). The carrier was added afterward in an adjustable phase, so that all three-component patterns of constant spectrum could be obtained by merely changing the phase of the central component. In general it was found that the tone quality was greatly affected by a change of phase. When the phase of the carrier was adjusted to give an amplitude modulation pattern the complex sounded quite harsh. The raucousness disappeared largely when the carrier phase was changed 90°, so as to approximate a frequency modulation pattern.

It is reported that a basic pitch, corresponding to the modulation frequency, was perceived. This pitch was most apparent when the waveform showed deepest amplitude modulation, corresponding to a raucous sensation. When the frequency modulation condition was approximated, the number of peaks per modulation period increased by a factor of two. The pitch tended to take a corresponding step, or faded away when the f.m.-signal did not show pronounced peaks.

The phase effects were found present when the modulation frequency is smaller than 40 per cent of the carrier frequency. The value of 40 per cent applies to sound levels of 60 db, it is stated that the critical separation decreases with decreasing level.

The dependence of the sound impression on the phases was considered to be caused by a limited resolving power of the ear. The bandwidths and response curves derived by Fletcher (F-5, F-3) were found adequate for the explanation. The pitch accompanying the peaked signals is in excellent agreement with the theory of Schouten for the residue. Finally, it is to be noted that Mathes and Miller report a basic pitch when the complex is inharmonic. They do not mention, however, whether this pitch deviates from the modulation frequency or not.

1.7. A further distinction of waveform and power spectrum of a sound was obtained by Miller and Taylor (MT-1). They used intermittent noise, made by passing a white noise through an electronic switch, driven by a square wave. The data on discrimination of the intermittency were found in agreement with the concept that the ear acts like having a constant decay time of auditory experience (Miller, M-1; Symmes and collaborators, SCH-1). They found also that the intermittent sound displayed a pitch-like character, corresponding crudely to the repetition frequency. It proved fairly difficult to determine this pitch, irrespective of whether a sine wave or a more complex signal was used for comparison.

The sensitivity for variations in the burst frequency proved to be rather high especially for the frequencies below 300 c/s. The deviation of the difference limen from the general course of the critical bandwidth curve (F-3, Z-2, Z-3, SGST-1) was considered to point toward a frequency principle as being responsible for the impression of pitch. In order to test this idea further the authors used binaural signals. To each ear an intermittent white noise was fed, with slightly different repetition frequencies. In this case binaural beats were reported, that is, a continuously changing localization of the sound, dependent upon the relative moments of the bursts in both ears.

1.8. Licklider considered the residue effect of Schouten as a very important effect for auditory theory. In an effort to fit both this effect and the pitch character of intermittent noise (section 1.7) into an extended place theory, a neural action roughly analogous to short-time autocorrelation of neural activity was postulated (L-1). The neural configuration is essentially two-dimensional. The first axis represents the result of the cochlear filtering action. The activity of each point along this axis is delayed and compared with the initial value in order to determine crudely the autocorrelation function. The behaviour of this function is projected along the second axis. Whenever a pronounced periodicity is present the patterns of activity show a marked similarity in their autocorrelation functions. According to the theory, this may account for the similarity of pitch in widely divergent sounds (Duplex Theory).

In later reports Licklider took the different view as expressed by Hoogland (section 1.3.) into consideration

too. The Schouten hypothesis was submitted again to a crucial test. The channels of reception, normally in use for low tones, were saturated by masking with a filtered noise. It turned out that the residue was not impaired, which proved that the residue phenomenon is solely concerned with those high tone channels that are occupied by the components of the signal. The explanation of Hoogland's negative results is covered in Licklider's lecture at the Third London Symposium on Information Theory (L-3). The argument is concerned with questions regarding the relative phases of the components (see section 7.3).

In order to clarify his ideas Licklider describes an experiment with eight separate oscillators covering eight harmonics of some fundamental. The oscillators are synchronized by a pulse series, each synchronizing signal being delayed by an adjustable time interval. Thus the phases of the harmonics can be controlled separately. Once adjusted, they remain constant and the signal retains its shape. Results are in short that the residue effect is the most prominent when the waveform approximates an impulsive pattern.

In the same paper an extension of Licklider's Duplex Theory on Pitch Perception (L-1) is given, an extension to binaural phenomena. A peculiar phenomenon found by Huggins in 1953, was the immediate cause of this extension (H-5).

In the experiment white noise is passed through an all-pass filter that introduces appreciable phase shift over a substantially limited range of frequencies. This filtering does not alter the impression of the noise, of course. Now the input signal is fed to one ear and the output to the other. Then a faint tone is heard amidst the noise, with a pitch corresponding roughly to the frequency region of phase anomaly. The auditory experience is not unlike that obtained by enhancing the frequency range under concern in the spectrum.

In order to explain this effect, Licklider added a third dimension to the two-dimensional pattern of neurons, implied by his "duplex theory" (L-1). This third dimension displays the time relations of the signals on both ears. In this way the frequency region, where the phases in both ears are unequal in Huggins' experiment, is spatially separated from the rest of the noise. Since this region is narrow it can give a tonal sensation on both place and autocorrelation properties. From this one can conclude for the moment that the abilities of the ear to perceive pitch are certainly not restricted to the monaural case.

Chapter II

EXPERIMENTAL METHOD

2.1. Though similar in the creation of a more or less marked periodicity or in the maintainance of constant frequency differences, the signals used in the experiments reported in Chapter I were obtained by quite different methods. Schouten used principally filtered pulse series with a broad spectrum. Fletcher used separately generated components and so did Hoogland. The sounds studied by Davis and collaborators and by Thurlow and Small can be classified under filtered pulse series with a narrow spectrum, while the modulation products of Mathes and Miller are to be placed somewhere between Schouten's and Hoogland's complexes. This classification, in order of decreasing accuracy of the relations between the components, is important as an aid in designing further experiments.

Sounds in which all components are harmonically related to some fundamental frequency will be called harmonic. When the frequency differences between adjacent components are perfectly equal, but the harmonic relation between the frequencies is lost, we will call the inharmonic complex coherent, since the separation of components is still derived from one source. When the exact relationship between components is lost, because of individual tuning of the tones, we will call the signal incoherent. Chopped noise obviously falls outside the scope of this classification. Signals like these and those consisting of narrow bands of noise instead of sinusoidal components may be called randomized.

2.2. The experiments on coherent inharmonic signals, quoted in section 1.3, form an attractive starting point for research. The pitch shifts reported, when measured quantitatively, can be expected to give a more detailed information about the properties of the residue. Instead of using a series of equidistant pulses, the spectrum of which is shifted in frequency, one can use a modulator for direct production of the desired complex. The procedure is essentially the same as that applied by Mathes and Miller (section 1.6), but with a different purpose. In order to obtain complexes with more than three components the modulation signal must consist of a number of harmonically related tones. The possibilities of this extended modulation method will be treated first.

2.3. When a carrier A of frequency a is modulated by a signal B consisting of the frequencies b_1, b_2, etc. the modulation result will contain components with frequencies:

$$a \pm b_1,\ a \pm b_2,\ a \pm b_3 \text{ etc. } ^{*)}$$

By setting up special relations between the input signals A and B, numerous kinds of complexes can be obtained. For instance, when B consists of harmonics of a frequency b (i.e. $b_1 = b$, $b_2 = 2b$, $b_3 = 3b$, etc.), the frequency differences of the components of the output signal will be equal (coherent tones). When in addition to this, the frequency b is synchronized by a, so that b is an integral fraction of a, all resulting components will be harmonics of the frequency b (harmonic complex).
When the carrier is shifted away from an integer multiple of b the signal becomes coherent inharmonic, thus including the signals used by Schouten and Fletcher. Incoherent complexes can be obtained by generating the components of signal B with separate tone generators. By application of noise, whether white noise or narrow band noise, to one of the inputs, numerous kinds of randomized signals can be obtained as well. It is to be concluded that the modulation method as discussed provides a useful method for the research intended. In this method all variables of a symmetrical spectrum are under control and thus the comparison between various kinds of complexes is facilitated and the inherent conditions for the creation of the residue can be investigated.

2.4. A coherent modulation signal is usually produced by filtering an equidistant pulse series. The pulse generator is triggered by a sine wave, the duration and

*) In some instances the component frequencies are considered as their algebraic values. This applies of course only to a "difference tone", the frequency of which will be represented by a combination like $a - b_j$.

the amplitude of the pulses being adjustable within wide limits. The pulse generator has been used also for verification of Schouten's observations (section 3.1) and for that reason it is provided with additional features. The phase of the pulses relative to the trigger signal can be adjusted. This is realised by adding a variable d.c. component to the trigger signal in order to shift the zero crossings. The pulse apparatus is built in triplicate, so that three pulses per cycle of trigger signal can, if desired, be produced.
The filter *) consists of a double set of octave filters in the range 100 to 25000 c/s, with an attenuation of 18 db per octave outside the pass band (third order Butterworth filter). The frequency characteristics are shown in Fig. 2.4.1. It was verified that parallel connection of different filters did not produce marked cross-over effects, so that wider pass bands could be obtained as well (Fig. 2.4.2). For the purpose of sharp attenuation filters of both sets could be cascaded directly. This proved to be very effective indeed (Fig. 2.4.3).

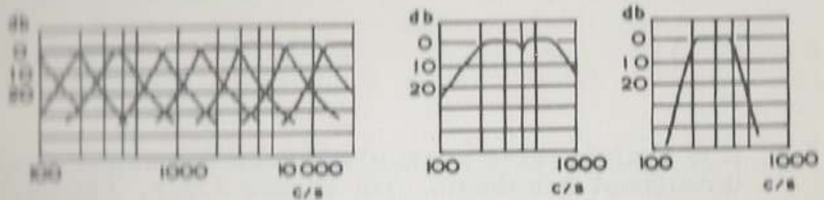


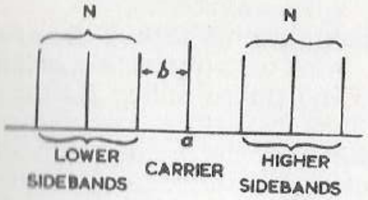
Fig. 2.4.1 (Left) Frequency response of octave band filters.
Fig. 2.4.2 (Centre) Frequency response of two adjacent filters parallel connected.
Fig. 2.4.3 (Right) Frequency response of two identical filters in cascade.

The modulator consists essentially of four triodes connected in such a way that both carrier and modulation signals are cancelled and only the modulation products

*) Dr. Schouten kindly provided us with this filter.

are present in the output (see section A.3 of Appendix). With the help of potentiometers the system can be adjusted for suppression of the input signals. When desired, a proper amount of carrier can be added to the output.

The signals are amplified by a high quality amplifier and finally fed to a loudspeaker system with a reasonably flat frequency characteristic.



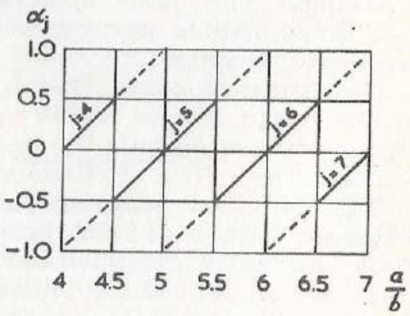


Fig. 2.5.1 Spectrum of typical signal

Fig. 2.5.2 Inharmonicity index versus a/b.

2.5. It is useful to give a list of symbols and names used throughout this thesis. See Figure 2.5.1. The carrier frequency a will be called c e n t r e f r e q u e n c y. The angular velocity $2\pi a$ will be denoted by ω in the calculations. The modulation signal B will be called b a s i s s i g n a l. This signal is in most cases harmonic. It will then have a fundamental frequency b. This is called the b a s i s f r e q u e n c y. Its associated angular velocity $2\pi b$ is represented by μ. The number of components of the basis signal shall be N, hence the number of components of the signal is 2N+1.

An inharmonic complex can be considered as derived from a harmonic one by a centre frequency shift over d cycles per second. The i n h a r m o n i c i t y i n d e x α is defined as the ratio of d and b:

$$\alpha = \frac{d}{b}$$

Since a given complex can be derived in this way from several harmonic complexes, differing with respect to the ratio j of centre and basis frequencies, the value of d is dependent upon the choice of j. The corresponding inharmonicity indexes can be distinguished by adding a subscript to the symbol. For instance, α_j is used when the given complex is compared to a harmonic complex, the centre frequency of which is the jth harmonic of the basis frequency. Hence it follows that

$$\alpha_j = \frac{a}{b} - j.$$

The number k shall be used for the value of j that yields the smallest magnitude of α_j. Figure 2.5.2 shows the course of α_j as dependent upon a/b.

An incoherent complex can be compared to a harmonic complex with the same parameters b, a, and N. The components to be compared differ only in phase. A convenient way for expressing those phases will be introduced in section 4.2.

Chapter III

EXPERIMENTS I

3.1. In order to get acquainted with the particular tone character of the residue it is helpful to repeat the experiments of Schouten with the pulse generator alone. The pulse series is combined with the trigger signal, the former in such a phase that the fundamental can be suppressed by proper adjustment of the amplitudes. The principal feature that the pitch of the sound does not change when the fundamental is suppressed is confirmed. By incomplete suppression of the fundamental two components with the same pitch, the fundamental and the residue can be heard simultaneously, just as is described by Schouten. When now a pure tone of nearly the same frequency as the complex is added, the beats are confined to the pure tones, i.e. the fundamental and the extra tone. In the case the fundamental is suppressed totally, these beats disappear altogether, again verifying that Schouten's view was correct.

Any irregularity of frequency which, according to Hoogland, can cause results like these, is unlikely to occur with the purely electronic apparatus used. Such a variation of frequency (rather of phase) can be deliberately introduced by adding some alien signal, e.g. a noise, to the trigger signal. It is found that this is easily detected by the ear, to such an extent that the tonal stability of the sound is severely impaired.

The experiments with two pulse series of the same frequency, but different "phases", yield the same results as described by Schouten, namely an octave jump when the pulses of one series are shifted with respect to the points midway between the pulses of the other. In order to suppress any fundamental, both series of pulses are combined with the triggering tone, in such a way that neither of the series contains the fundamental as Fourier component.

An extension is made to the case of three pulses per cycle of the trigger tone. When the pulses are equidistant, the pitch corresponds to the third harmonic of the

trigger tone. When one or more pulse series are shifted in time, the pitch drops a duodecime (an octave plus a fifth) to the value corresponding to the new periodicity. A pitch jump of a fifth is the most apparent, thus revealing that the chroma intervals (Révész, R-2; Bachem, B-1, B-2, B-3; Licklider, L-1) are easily determined from a residue.

3.2. With the help of the modulation process described in Chapter II (section 2.3) complexes of a few equidistant tones are easily set up. When starting with a frequency of 200 c/s, several harmonics of this frequency can be produced when:

$$a = 2000 \text{ c/s},$$
$$b_1 = 200 \text{ c/s},$$
$$b_2 = 400 \text{ c/s},$$
$$b_3 = 600 \text{ c/s}.$$

The given combination will lead to a complex consisting of the frequencies:

1400 - 1600 - 1800 - 2000 - 2200 - 2400 - 2600 c/s... (1)

They produce together a striking impression of a pitch of 200 c/s, *) even when the sound is weak (normally sound levels of 20-40 db were used). When instead of 2000 c/s another multiple of 200 c/s is chosen as centre frequency, again a combination of harmonically related frequencies will result. For instance, a = 2200 c/s produces with the same basis signal:

1600 - 1800 - 2000 - 2200 - 2400 - 2600 - 2800 c/s... (2)

The residue is again 200 c/s. The tone qualities of the sound complexes (1) and (2) are very much alike, in fact these sounds are almost indistinguishable from one another.

3.3. The effect indicated by Schouten can now be studied by gradual variation of the centre tone. We will restrict ourselves to the interval 2000-2200 c/s for a. When starting on the lower side (2000 c/s) a small increment of the centre frequency causes the pitch of the

*) The distinction between pitch level and frequency will not be maintained. As a consequence pitch will be often given in cycles per second.

idue to rise, without any loss of smoothness, however. For instance, when a centre frequency of 2030 c/s is chosen, and the complex consists of the following frequencies:

0 - 1630 - 1830 - 2030 - 2230 - 2430 - 2630 c/s, . . (3)

pitch is definitely higher than 200 c/s. On rough comparison with a pure tone, or preferably, a filtered pulse ies, it is found to be about 205 c/s. (The exact determination of pitch associated with inharmonic complexes be treated in chapter V. Here we will discuss only litative results).

When the process of carrier shift is started at the other of the interval considered, i.e. 2200 c/s, and the tre frequency is decreased, the pitch is found to drop. ce on both sides of the interval the pitch corresponds 00 c/s, some kind of paradox is indicated. With continous variation through the interval it is observed that ewhere in the centre of the interval the attention tches over to another pitch. Obviously two pitches possible in this region, and it depends on the direc- of carrier shift, which one the mind is focused upon. is interesting to consider the exact centre point of interval somewhat nearer. The associated complex (a = 2100 c/s, see Fig. 3.3.1).

0 - 1700 - 1900 - 2100 - 2300 - 2500 - 2700 c/s. . . (4)

it obviously consists of odd harmonics of 100 c/s. n by the most careful listening no trace of a residue 00 c/s is found. The two pitches reported above are to 200 c/s and are continuous extensions of the pitch- round the end points of the interval.

igure 3.3.2 depicts the pitch variation in the interval sidered. This kind of behaviour can be found in every re-tone interval between two successive multiples 00 c/s. Only when the frequencies involved become high, the pitch shifts are too small to be detected and ddition to this the tonality becomes rapidly weaker.

3.4

Once it is shown that an inharmonic complex can give rise to a residue, the pitch of which does not espond to the difference frequency, some estimate be made of the expected pitch shift as dependent the carrier-frequency shift. In the harmonic case

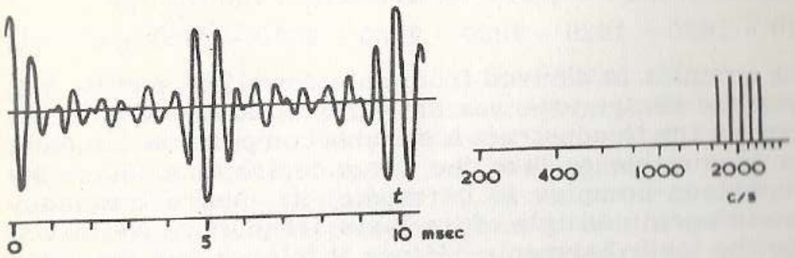


Fig. 3.3.1 Waveform (left) and spectrum (right) of a signal consisting of odd harmonics only.

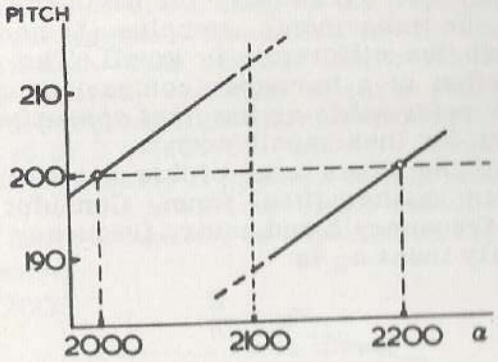


Fig. 3.3.2 Pitch versus centre frequency (b = 200).

the frequencies are multiples of the frequency that corresponds to the residue. When the complex becomes inharmonic, the exact relation is lost. In other words there is no frequency in the neighbourhood of the basis frequency, of which the presented frequencies are exact multiples. It seems logical to assume that the residue corresponds to such a frequency that its correct integral multiples resemble the given tones as much as possible.

This can be clarified with a numerical example. Consider the complex (3) of section 3.3:

1430 - 1630 - 1830 - 2030 - 2230 - 2430 - 2630 c/s... (1)

This complex is derived from complex (1) of section 3.2, where the centre tone was the tenth harmonic of 200 c/s. When we try to construct a suitable comparison complex, it is reasonable to take the same centre tone. Since the comparison complex is harmonic, its centre frequency is to be an integral multiple of its basis frequency. We take it to be the tenth harmonic. Hence it follows that the basis frequency is 203 c/s. The comparison complex consists of the following frequencies:

1421 - 1624 - 1827 - 2030 - 2233 - 2436 - 2639 c/s... (2)

and its residue pitch is 203 c/s. The similarity between (1) and (2) is indeed very great. These complexes can immediately be compared experimentally, since they differ in only one parameter: the basis frequency. It is found that the inharmonic complex (1) has the higher pitch, though the difference is small. The pitch, found by construction of a harmonic comparison complex like (2), will be referred to as the *first approximation* of the residue pitch for inharmonic sounds.

In view of the importance of this derivation it will be expressed in mathematical form. Consider a complex with basis frequency b and centre frequency $kb + d$. The inharmonicity index α_k is

$$\alpha_k = \frac{d}{b}$$

In first approximation the pitch is that of a harmonic complex with equal centre frequency. The basis frequency is then

$$b' = \frac{kb + d}{k} = b + \frac{d}{k}$$

or expressed in relative value

$$\frac{b' - b}{b} = \frac{d}{kb} = \frac{\alpha_k}{k}$$

The relative pitch shift β is thus expressed by

$$\beta = \frac{\alpha_k}{k}$$

It is easy to verify that the so-derived pitch is independent of the basis frequency under the condition of constant centre frequency.

3.5. From the comparison it is concluded that the pitch depends also on the basis frequency. This feature is explored further. For the sake of numerical simplicity we will take 1800 c/s as the centre frequency.

When the basis frequency is increased from 200 c/s, the residue pitch drops, compared to the initial level of 200 c/s. This trend of variation, contrary in direction to the basis frequency, leads again to a paradox similar to that considered above. Associated with $b = 225$ c/s is a harmonic complex, giving rise to a residue of 225 c/s, (because $1800/8 = 225$). When the basis frequency is decreased from this value, the pitch rises above 225 c/s. In the interval 200 to 225 c/s two conflicting trends are thus to be noted. On closer examination it turns out that the attention switches over to the other pitch, whenever a critical region somewhere in the centre of the interval is passed. This region, where one of two pitches can be perceived, contains again a point that corresponds to a complex of odd harmonics only. That point is determined by $b = 1800/8{,}5$ and the components corresponding to this basis frequency will be odd harmonics of $1800/17$ c/s. The behaviour of the residue in dependence on the basis frequency is depicted in Fig. 3.5.1.

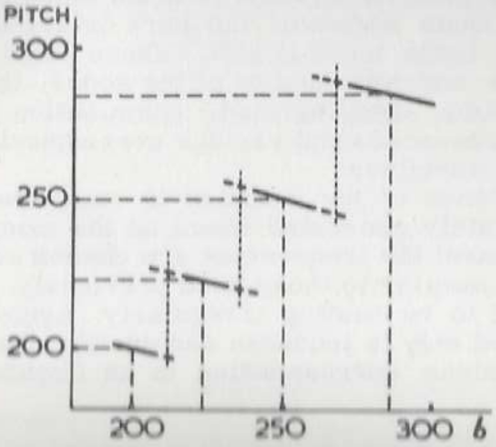


Fig. 3.5.1 Pitch versus basis frequency (a = 1800).

We can conclude that the residue pitch is for small inharmonicities nearly independent of the basis frequency. The pitch of an inharmonic complex is approximately equal to the nearest basis frequency which is harmonically related to the centre frequency. This is equivalent to the statement that the „first approximation" of the residue pitch is a rather accurate measure of it.

3.6. The absence of beats in coherent inharmonic complexes deserves a short discussion. When all components of a harmonic complex are shifted 1 c/s the waveform has a repetition period of one second. The experiments show that "inharmonicity" beats corresponding to this period do not arise at all. This fact is one of the central questions in the residue problem.

Apparently, the absence of beats in the sound is related to the coherence of its components, this being the only remaining relation of the frequencies. This feature is lost altogether when the complex is made up of separately generated components. From the results of Mathes and Miller it is to be expected that in this case beats will arise.

As an intermediate step we will retain the modulation method and use separate tones for constituting the basis signal (incoherent basis). Modulation with one frequency produces a weak and almost indiscernible residue. When a second tone, the octave of the first, is added to the modulation signal, the residue is much more pronounced. It shows beats whenever the pure octave relationship between the basis tones is lost. These beats cover only the loudness and tone quality of the sound, the pitch remains the same. Oscillographic examination shows that the most pronounced tonal residue corresponds generally to a peaked waveform.

The coherence of the complex is completely lost by using separately generated tones as the components. In this experiment the frequencies are chosen such that the spectra are similar to those used previously. The residue is heard to be beating irregularly. Again it is found to be affected only in loudness and quality, the most pronounced residue corresponding to an impulsive waveform.

3.7. In some cases a coherent inharmonic complex does produce "inharmonicity beats". Careful examination shows that this is caused by an inconstant amplitude of one or more components. These amplitude variations are the result of beats between the component under concern and an alien tone of nearly the same frequency. There are two causes of production of these unwanted tones.

The first is incomplete suppression of the basis signal, especially its transients which can sometimes survive the filtering action. Careful adjustment of the suppression eliminates the additional components and the beats of the residue.

The second cause is a more fundamental one. When the highest basis component has a frequency exceeding that of the centre tone, modulation products with negative frequencies *) will arise. At conditions of inharmonicity these components beat with those having positive frequencies because the absolute values of frequency are different. These two causes of inharmonicity beats can thus be classified as artifacts. Elimination of both reveals that an inharmonic complex does not show beats and is on listening not distinguishable from a related harmonic complex other than by its pitch.

*) The reader is referred to the note on page 22 (section 2.3) for the algebraic values of frequency.

Chapter IV

DISCUSSION I

4.1. The qualitative experiments on the pitch of inharmonic signals prove that the pitch of the residue does not always correspond to the common difference frequency. This result shows that the residue is not a distortion product. In this respect the basic evidence obtained by Schouten (section 1.2) and the additional proof given by Licklider (section 1.8) are confirmed. Nevertheless, it is interesting to consider for a moment the process of quadratic distortion and the correspondence between difference tone and residue.

The presented signal is written as

$$f(t) = \left\{ 1 + B(t) \right\} \cos \omega t \; \ldots\ldots \qquad (1)$$

in which $B(t)$ denotes the basis signal of frequency b and $\omega = 2.\pi a$.

The quadratic term yields

$$f^2(t) = \left\{ 1 + B(t) \right\}^2 \left(\tfrac{1}{2} + \tfrac{1}{2} \cos 2\omega t\right) \; \ldots\ldots \qquad (2)$$

The component with frequency b arises solely from the term

$$\tfrac{1}{2}\left\{ 1 + B(t) \right\}^2$$

provided that the highest component of $B(t)$ has a frequency smaller than $a - \frac{1}{2} b$.

The amplitude of the difference tone is thus independent of the carrier frequency a. This implies that the difference tone is not affected by inharmonicity. The residue displays the same property.

The amplitude of the difference tone depends upon the phases of the components of $B(t)$. It is maximal when these are such that all components can be written as

cosine functions with zero initial phase angle. The residue shows a similar trend when the phases are changed.

The pitch of the residue cannot be explained in terms of quadratic distortion.

4.2. The following attempt at better understanding the experimental results is based upon separation of pitch and phase effects in relation to different aspects of the waveform. Since modulation is multiplication of the input signals the instantaneous amplitude of the output signal is proportional to the value of the basis signal plus a constant. The waveform of the basis signal thus appears as the envelope of the resulting waveform (see Fig. 4.2.1 and compare formula 1 of section 4.1).

The phase relations of the basis signal are retained in the shape of the envelope. Since variations of these phase relations is found to affect the tone quality of the residue one concludes that the timbre is closely related to the envelope. A change of the centre frequency affects the pitch but it does not change the tone quality nor does it give the sound a fluttering character. Such a variation does not alter the envelope. The pitch of the residue then seems related to the "fine-structure" of the signal (the fast oscillations between the limits set by the envelope).

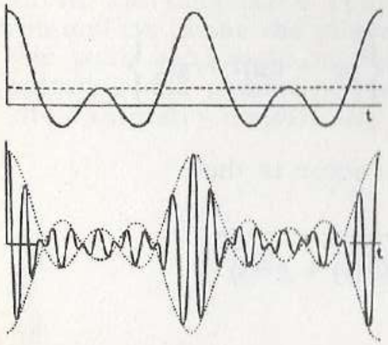


Fig. 4.2.1 Basis signal (above) and modulated signal (below).

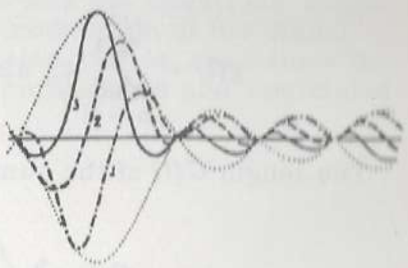


Fig. 4.2.2 Waveforms in three successive basis periods.

In Fig. 4.2.2 the waveforms in successive periods of the basis signal are plotted in one graph. The envelope is seen to be tangent to the composite picture. When the phase of the centre tone is taken as a parameter that generates a set of curves, the envelope obeys the usual mathematical definition. The envelope is a generalized amplitude of the signal.

The envelope can be calculated as follows.

The signal is written as

$$f(t) = \sum_{n=-N}^{+N} c_n \cos\left\{(\omega + n\mu)t + \psi_n\right\} \ldots \qquad (1)$$

where $f(t)$ is the instantaneous value, $\omega = 2\pi a$, $\mu = 2\pi b$ (see section 2.5), c_n is the amplitude and ψ_n the phase of the component with frequency $a + nb$.

The components can be represented as the projections of rotating vectors on a fixed projection axis. The complete signal is then the projection of the sum of these vectors on the same axis. The length of the sum vector is the "amplitude" of the signal (compare Stumpers, S-8).

The component of the sum vector perpendicular to the projection axis is

$$g(t) = \sum_{n=-N}^{+N} c_n \sin\left\{(\omega + n\mu)t + \psi_n\right\}$$

The length $C(t)$ of the sum vector is thus

$$C(t) = \sqrt{f^2(t) + g^2(t)}$$

When the vectors representing the components are projected on a coordinate system that rotates with an angular velocity ω the length of the sum vector is found to be

$$C(t) = \sqrt{\left\{\Sigma c_n \cos(n\mu t + \psi_n)\right\}^2 + \left\{\Sigma c_n \sin(n\mu t + \psi_n)\right\}^2} \ldots \qquad (2)$$

The envelope is thus found from the magnitude of the complex function $c(t)$ defined as

$$c(t) = \sum_{n=-N}^{+N} c_n \exp i(n\mu t + \psi_n) = C(t) \exp i\psi(t)$$

In many instances the envelope is easily obtained graphically from the vector representation of $c(t)$.

The original signal (1) can be rewritten as

$$f(t) = c(t).\exp i\omega t + c^*(t).\exp(-i\omega t)$$

where $c^*(t)$ is the complex conjugate of $c(t)$.

In terms of the amplitude $C(t)$ and the phase $\psi(t)$ of the complex function $c(t)$ the signal (1) can be expressed by

$$f(t) = C(t).\cos\left\{\omega t + \psi(t)\right\} \ldots \qquad (3)$$

The complex function $c(t)$ is a generalized modulation function. Its magnitude represents the amplitude modulation and its phase the phase modulation of the signal.

For pure amplitude modulation $c(t)$ is real since the amplitudes and phases of the components are restricted by the symmetry conditions:

$$\begin{cases} c_{-n} = c_{+n} \\ \psi_{-n} = -\psi_{+n} \end{cases}$$

4.3. An increment of the frequency of a pure tone is equivalent to the addition of a phase angle that increases proportionally with time. A slightly inharmonic signal can thus be regarded as a harmonic complex in which the phase of the centre tone varies continually.

Different phase relations between the centre and basis tones alternate.

Experimentally, loudness and tone quality are constant. Loudness and tone quality are thus independent of the phase relation between centre and basis tone. This property is based upon experiments on pure amplitude modulation.

For a general type of signal the phases ψ_n (section 4.2) are no longer restricted by symmetry relations.

When the independence mentioned above applies to these signals, the phases of the components can be increased by a constant amount without changing the tone quality. By a variation of the time-reference point the phases are increased by angles that are proportional to the frequencies of the components.

Both possibilities are included in the statement that *a linear function of the frequency can be added to the phases without change of tone quality.*

Hence the phases can be normalized so that two of them become zero.

Whether this procedure is admissible is still a question. In the experiments the acoustic signal was produced by a loudspeaker, therefore, large phase deviations occurred. This suggests that it is admissible to extend the results to more general signals.

However, it is necessary to corroborate such a conclusion.

4.4. For further experiments a greater versatility of the apparatus was desirable. Accordingly, a unit similar to that used by Fletcher and by Schouten in their experiments on inharmonic signals was constructed, consisting essentially of two modulators and a sharp band pass filter.

The first modulator modulates the signal on a high frequency carrier, the filter suppresses the lower sidebands and the resulting signal is demodulated by the second modulator.

The modulators are of similar construction as the one used in the experiments described. The filter has a pass band from 60 to 64 kc/s *), and the oscillators providing

*) This filter was again put at our disposal by Dr. Schouten as is gratefully acknowledged here.

the modulation and demodulation frequencies are adjustable in the range from 55 to 70 kc/s.

With this unit the components of a signal can be shifted in frequency over a constant amount. The unit can perform additional functions like modulation with partial sideband suppression and heterodyne filtering.

When used as a modulator the suppression of the basis signal is improved. This is advantageous in exploring "low" complexes. The modulators can, if desired, be used separately.

4.5. Also in the experiments of section 3.6 a loudspeaker was used for the sound production.

Although the frequency characteristic shows an irregular shape (mostly due to room acoustics) it is remarkable that a close correspondence between electric input and subjective impression is indicated. The most pronounced residue is usually produced when the electric signal shows an impulsive pattern.

It is, of course, necessary to check this correspondence by measuring the actual sound pressure at the ear.

More easily, a high quality earphone can be used. This eliminates the sound measurement because the substantially flat transmission makes the sound wave a nearly exact replica of the applied voltage (see section A.7 of the appendix).

In all further experiments such an earphone has been used.

Chapter V

EXPERIMENTS II

5.1. Accurate determinations of the pitch of the residue can be made by comparison with a signal of known pitch. It is to be desired that the two sounds are alike in quality, i.e. as comparison signal a residue is to be preferred. In order to have a well-defined comparison pitch a harmonic complex is used. Its pitch level will be taken equal to the fundamental frequency.

Such a harmonic signal can be obtained by synchronizing the basis frequency with the centre tone.

A time-base circuit as is used in oscillographs will suffice for this purpose. However, it is hard to synchronize with a high harmonic over an appreciable frequency range.

For this reason it was decided to use a frequency divider instead of a time-base or multivibrator circuit for generation of the basis signal.

The circuit operates as follows.

Each cycle of the input signal a charge is put on a capacitor. When the voltage developed across this capacitor exceeds a preset threshold value the capacitor is discharged rapidly and the process of charge by piecemeal is repeated over and over again (Fig. 5.1.1). The threshold determines the ratio by which the frequency is divided.

The output of this divider resembles a time-base signal. The amplitudes of the Fourier components of a pure sawtooth wave are inversely proportional to their frequencies. This relation holds for our signal except for the components of the input frequency and its multiples, which are reduced in amplitude.

Differentiation by means of an R-C network makes the first few harmonics of equal amplitude.

The signal is then filtered by two third-order filters in cascade in order to impose a sharp limit on the number of components of the basis signal.

The frequency divider is not only used for the generation of the standard signal but also for the inharmonic complex under study.

A block diagram of the set-up for pitch determination is pictured in Fig. 5.1.2.

By means of the pushbotton labelled (X) the tone generator A is connected to the modulator M to provide the carrier, and in addition the tone generator B is made to provide the basis signal by way of the frequency divider D and the filter F.

The pushbutton (S) connects the third tone generator C to both the carrier input of the modulator and the input of the frequency divider in order to create a harmonic comparison complex.

Since but one tone generator (Radiometer) with a calibrated dial for frequency increments is available, this unit is used as the C-generator.

The frequencies of A and B are adjusted by oscillographic comparison with C. After adjustment the signals (X) and (S) are presented alternately. The listener then adjusts C until the pitches of the two sounds are heard equal.

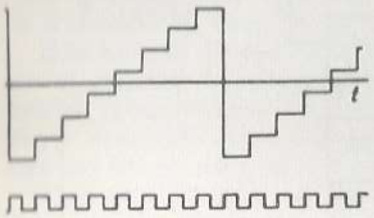


Fig. 5.1.1 Operation of the frequency divider (trigger signal as reference).

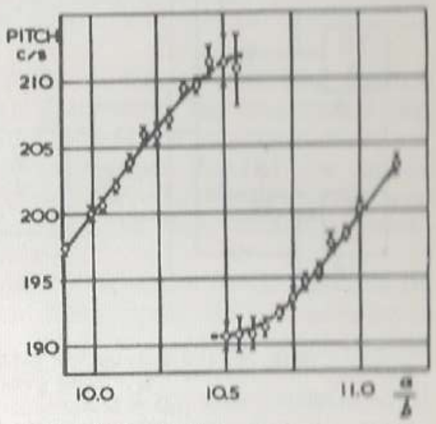


Fig. 5.2.1 Pitch versus centre frequency (b = 200)

The number of components is five, this being the lowest number from which a stable and distinct residue can be obtained.

For the electroacoustical conversion a Permoflux earphone type PDR-10 was used (see Appendix, section A.8). The earphone is connected via 10 kΩ to the final amplifier in order to avoid any possibility of damaging the earphone during the adjustment. The voltage across the output of the amplifier is measured and analysed by a wave analyser. The centre tone and the first-order sidebands are adjusted to 30 db above the threshold for 1000 c/s of the subject. The second-order sidebands are usually less than 2 db weaker.

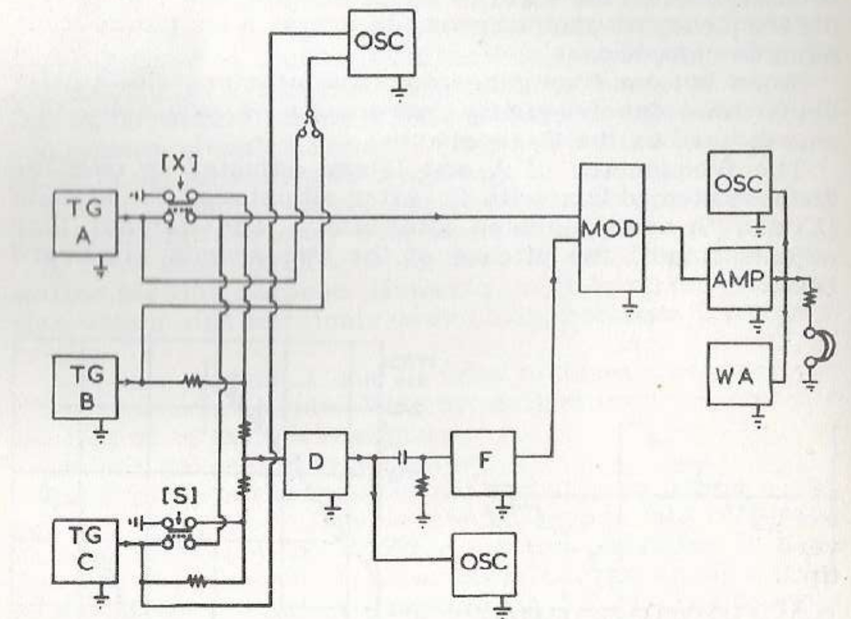


Fig. 5.1.2 Set-up for pitch measurements.
TG = tone generator, MOD = modulator,
D = frequency divider, F = filter,
OSC = oscillograph, AMP = amplifier,
WA = wave analyser.

5.2. The limited number of pass bands of the filter used for the basis signal sets a limit to the number of basis frequencies, when one wants to keep the number of components of the basis signal constant.

Fig. 5.2.1 depicts a typical result obtained for the pitch as a function of the centre frequency.

The distribution of judgements indicates a nearly linear dependence of pitch on centre frequency.

The second feature is the gradual widening of the distribution when the point half-way the interval is approached. It is very difficult to judge pitch in this region because the attention switches suddenly during measurement.

It seems that, whenever the pitch of the comparison signal approaches one of the two possible pitches of the residue, the attention is forced to switch to the other one.

Fig. 5.2.2 depicts the course of the residue pitch in the whole centre-frequency range for a basis frequency of 200 c/s. The dotted lines correspond to the "first approximation" as described in section 3.4.

For the higher centre frequencies the judgements fail to be consistent. Just before the end point the judgements sometimes tend to separate into clusters corresponding to different curves.

It is suggested that the effect is due to a wrong choice of periodicity by the ear (see Chapter VIII, section 8.5). It is nearly impossible to investigate this effect further, as it cannot be elicited intentionally. For a few other basis frequencies from 100 to 400 c/s the results are very similar when the pitch scale of Fig. 5.2.2 is multiplied by an appropriate number.

The dependence on the basis frequency is measured by essentially the same method. Since greater variations of basis frequency are involved the basis signal is produced in a different way. The differentiated output of the frequency divider is filtered so as to obtain the fundamental only. This sinusoid is fed to both inputs of a separate modulator that produces frequency doubling. A combination of the basis tone and its octave is then used as the basis signal. Fig. 5.2.3 depicts a typical result of the measurements.

5.3. The amplitudes and phases of the components of signals obtained by amplitude modulation have cer-

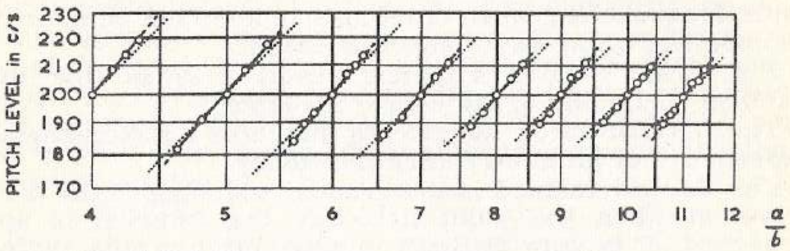


Fig. 5.2.2 Pitch versus centre frequency (b = 200).
Each point mean of 5 judgements.

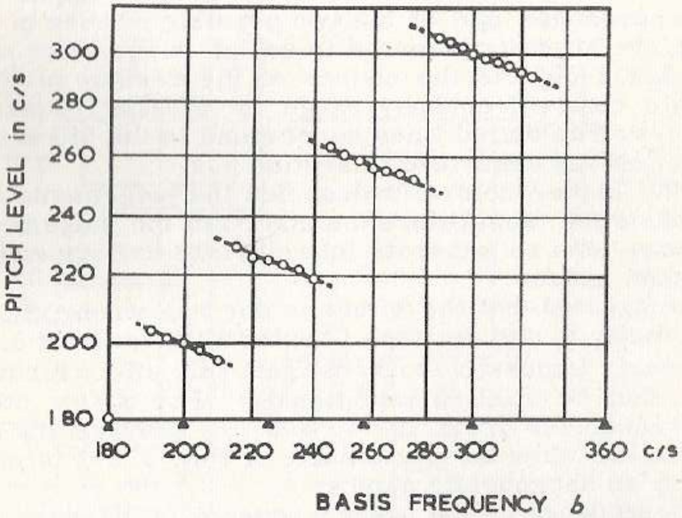


Fig. 5.2.3 Pitch versus basis frequency (a = 1800).
Each point mean of 5 judgements.

tain properties of symmetry (section 4.2). This correspondence is lost when a part of the sidebands is suppressed by a filter due to the non-constant delay of the filter near the region of cut-off. In this way more general phase conditions can be obtained. The sideband suppression is provided by the band pass filter of the double modulator unit. The second modulator carries the spectrum

back to the audible frequency range. Experiments with the so-obtained signals yield the result that the signals have exactly the same properties as the pure amplitude modulation products used in previous experiments.

The dependence of the pitch upon the centre frequency given by the data of section 5.2 applies to the more general signals. It must be noted, however, that the pitch matches involving signals without pronounced peaks are slightly less consistent. When tonality is defined as the reciprocal of the standard deviation of pitch judgements, the tonality is maximal when the residue has the sharpest timbre.

5.4. The deviations of the residue pitch from the "first approximation" (see section 3.4) may be due to some asymmetric weighting of the components.

When the higher components do not contribute as much as the lower ones to the determination of pitch the higher pitch shifts of the inharmonic signals can be explained.

In that case the effective centre tone is lower than the actual one. Its frequency shift, though in absolute value equal to the carrier shift, is in relative measure greater than the corresponding carrier shift and the pitch shift will be greater than expected.

When this is true the pitch of an inharmonic complex will depend on the width of the spectrum.

In Fig. 5.4.1 this effect is depicted. The number of components is determined by the setting of the basis-signal filters. A great number of determinations was needed in order to obtain sufficient accuracy. This effect, though not proving the above explanation, makes it highly probable.

5.5. We turn now to the phase effects indicated in section 3.6. It was shown that incoherent signals show variations in loudness and quality. Loudness and quality are thus dependent on the phases of the components (see section 4.3).

We now try to investigate this effect more closely. The experiments will be performed in such a way that different phase relations alternate. This is the easiest way to observe the phase effects.

First the experiment with an incoherent two-component basis signal is repeated in order to investigate the correspondence between tone quality and envelope shape of the signal as it enters the ear.

The instantaneous phase relation of the basis components is determined from the Lissajous pattern of the basis components. The phases of the signal components can be derived easily from the phases of the basis components.

The result confirms the prediction that the residue is loudest and sharpest when the phases of the outer sidebands are zero. This condition involves an envelope with the most pronounced peaks.

It is observed that the phase effects disappear when the ratio of basis and centre tone is increased beyond a certain value. The limiting points (basis-phase limits) where they become indiscernible are given in Table I.

Table I (section 5.5)

Basis frequency b	Critical centre frequency a	Ratio $\frac{b}{a}$
100	440	0,23
150	670	0,22
200	1000	0,20
300	1700	0,18
400	2000	0,20

5.6. We will now study the phase changes of the centre component. The simplest case is a three-component complex as used by Mathes and Miller (section 1.6). The set-up is shown in Fig. 5.6.1. A tone a of e.g. 1600 c/s is modulated by 200 c/s (b). The carrier is suppressed and replaced by an alien tone a' of, say 1600,5 c/s. The latter will be called pseudo-carrier.

All components are adjusted to 30 db (referred to the threshold for 1000 c/s of the subject).

The beats are heard with a frequency twice the frequency difference between carrier and pseudo-carrier.

This shows that the beats are not caused by some incomplete suppression of the carrier.

Moreover, this suppression could be measured by the wave analyser to be more than 50 db relative to the sidebands.

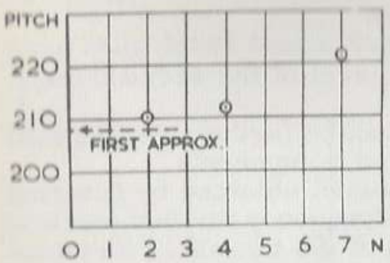


Fig. 5.4.1 Effect of the spectrum width (b=200, k=8, α_k=0,3). Each point mean of 20 judgements.

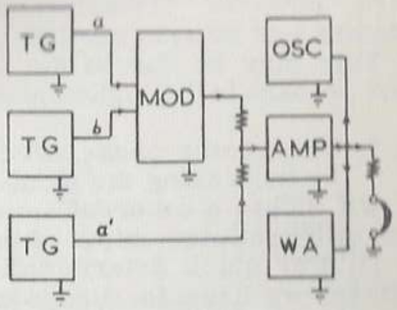


Fig. 5.6.1 Set-up for centre-phase effect (N=1). See legend to Fig. 5.7.1.

By oscillographic examination of the sum of carrier and pseudo-carrier it is found that the residue is most pronounced when they are in phase and when they are 180° out of phase. In both cases the waveform of the sound is of a pure amplitude-modulation character, thus confirming Mathes and Miller's finding and justifying the periodicity hypothesis of the envelope in section 4.2.

By a variation of the basis frequency it is found that the beats disappear when the separation between components becomes too great.

Table II shows these limits for some centre frequencies.

Table II (section 5.6)

Centre frequency a	Critical basis frequency b	Ratio $\frac{b}{a}$
600	110	0,18
800	144	0,18
1000	220	0,22
1600	320	0,20
2000	390	0,20

It is seen that the ratio of critical basis frequency to the carrier frequency is almost constant. The value differs, however, from the value of 40 per cent given by Mathes and Miller (section 1.6).

This may be due to the lower sound level used here and perhaps to the higher noise level of the surroundings.

5.7. The centre-phase effect can be further investigated by increasing the number of components.

We choose a coherent basis signal obtained by filtering the differentiated output of the frequency divider (section 5.1). For quick determination of the threshold for phase effects we have to change either the centre or the basis frequency. Since the carrier must always be replaced by the pseudo-carrier and the variations of the basis frequency are limited by the filter capabilities, there remains only a frequency shift of the whole complex by means of the double modulator.

Fig. 5.7.1 shows the block diagram of the set-up for this experiment. Modulator I produces the sidebands of the constant complex by modulating the carrier A (about 1500 c/s) with the basis signal while suppressing the carrier. By suitable connections of the filter F_1 and proper choice of basis frequency B/k, the basis signal is adjusted to consist of equal components and to have a peaked waveform.

A pseudo-carrier A' is added to the complex. The feature that the most marked residue arises when the envelope is peaked, is again found for the so-obtained signal.

At the moments that A and A' are in phase the envelope looks like Fig. 5.7.2-A. When A and A' are of opposite phase the waveform has changed to that of Fig. 5.7.2-B. These waveforms represent positive and negative modulation respectively.

The residue is loudest and sharpest at the moments, corresponding to the condition shown in Fig. 5.7.2-A. This is easily understood since here the peaks are most pronounced. The number of residue beats per second is thus equal to the frequency difference between A and A'.

Now this signal is modulated with a frequency f_1 of 60,5 kc/s by modulator II, so that the higher sidebands lie in the centre of the pass band of filter F_2 (60-64 kc/s). The envelope is thus retained.

Demodulation is performed by modulator III, with a

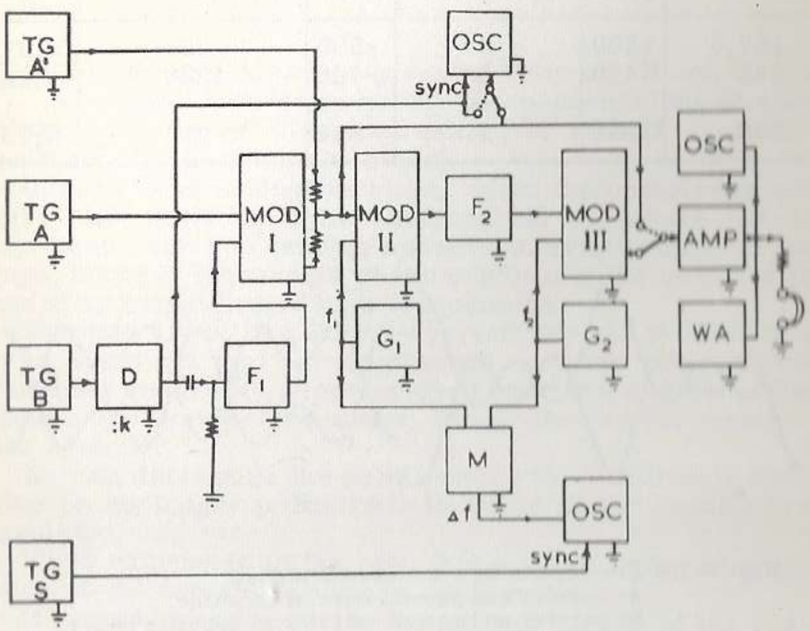


Fig. 5.7.1. Set-up for centre-phase effect (N>1).
TG = tone generator, MOD = modulator,
D = frequency divider, F = filter,
G = oscillator, M = mixer,
OSC = oscillograph, AMP = amplifier,
WA = wave analyser, S = standard for frequency measurement.

demodulation frequency f_2. The net result is that the spectrum has shifted towards the lower frequencies over an amount equal to $f_2 - f_1 = \Delta f$.

It is observed that increasing Δf leads to gradual disappearance of the beats. The centre frequency where the beats become indiscernible is tabulated in Table III for various basis frequencies.

Table III (section 5. 7)

Basis frequency b	A	number of components 2N+1	Δf	critical centre frequency a	ratio $\frac{b}{a}$
187,5	1500	5	-800	700	0,27
355	1420	5	-160	1260	0,28
230	1840	7	-930	910	0,25

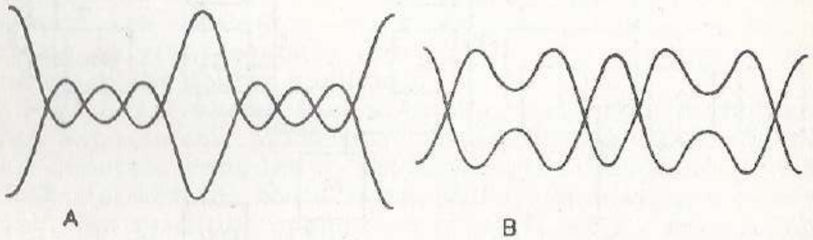


Fig. 5. 7. 2 Envelope of five-component signal.
A: carrier and pseudo-carrier in phase,
B: carrier and pseudo-carrier in opposite phase.

On comparison with table II it follows that the phase-bandwidth ratio b/a is somewhat larger for complexes of more than three components.

This may be due to the sharper peaks associated with the wider complexes, which lead to easier detection of beats.

In all these experiments on phase effects the residue is sharpest when the waveform shows the most pronounced peaks. A still more pronounced residue is found when the "fine-structure" during the peak of the envelope is out of phase with that between the peaks.

On the one hand this may point toward a mechanism with some inherent damping (MM-1). On the other hand it should be remembered that at the same time the centre

component reduces to the same order of amplitude as the sidebands, thus a clear residue is produced which is not masked by one of the components.

It is finally stressed that the effects described are independent of the harmonicity or inharmonicity of the signals.

5.8. At the critical points given in Tables, I, II and III the residue loudness is nearly independent of the phases of the components. The impression is then of a continuous sound (continuous residue).

In the region of degeneration, when the complexes are very high relative to the fundamental the phase effects are such that the residue appears intermittently. In the gaps between favourable phase conditions the complex is heard as consisting of high frequencies.

When the sound is gradually varied between these limits it is observed that the continuous and the intermittent residues sound like superimposed upon one another. The higher the complex becomes, the weaker is the continuous residue.

We can determine the points where the continuous residue is no longer perceived. In Table IV the results are tabulated.

When expressed in the ratio b/a a remarkable consistency of the determinations is again noted.

It seems safe to describe the various stages of the residue with the rank number of harmonics involved.

Table IV (section 5.8)

Limit of continuity

Set-up of section	Effect under study	Basis frequency b	Centre frequency a	Number of components 2N+1	Ratio $\frac{a}{b}$
5.5	Basis-phase effect	100	800	5	0,13
		150	1100		0,14
		200	1700		0,12
		300	1950		0,15
		400	2500		0,16
5.6	Centre-phase effect	140	800	3	0,17
		160	1000		0,16
		240	1600		0,16
		300	2000		0,15
		400	3000		0,13
5.7		187,5	1400	5	0,13
		355	2150	5	0,16
		230	1500	7	0,15

Chapter VI

EXPERIMENTS III

6.1. The separation of continuous and intermittent residue implies not only a difference in their time courses for incoherent signals. A difference in perceived quality is present as well.

The continuous residue sounds smooth but is of weak tonality, the impression being that the complex is nearly resolved into its components.

The intermittent residue is of a sharper character, both in timbre and tonality.

All evidence indicates that the latter kind of residue is brought about by a peaked waveform.

One may imagine that the continuous residue can be elicited by the use of a signal without peaks, yet containing equally spaced Fourier components.

Such a signal can be produced by frequency modulation. When the modulation frequency is b and the maximum frequency deviation is Δa, the modulation index

$$m = \frac{\Delta a}{b}$$

determines the distribution of the spectral components (v. d. Pol, P-1).

The frequency-modulated signal with mean frequency a can be expressed by

$$f(t) = c_0 \exp i(\omega t + m \sin \mu t)$$

$$(\omega = 2\pi a, \ \mu = 2\pi b)$$

The factor $\exp i\, m \sin \mu t$ is expanded into a Fourier series:

$$\exp i\, m \sin \mu t = \sum_{n=-\infty}^{+\infty} J_n(m).\ \exp i n \mu t$$

The coëfficients are Bessel functions of order n. The Fourier analysis of f becomes:

$$f(t) = c_0 \sum_{n=-\infty}^{+\infty} J_n(m) \exp i\ (\omega + n\mu) t$$

From the course of the Bessel functions (Jahnke-Emde, JE-1) it is seen that for a modulation index slightly less than unity the carrier component is about twice as large as the first sidebands and the second sidebands are much smaller.

A corresponding condition is set up experimentally by feeding a pure-tone signal to the reactance tube of the Radiometer tone generator.

The carrier is adjusted at 31 db, the first-order sidebands being 25 db and the second-order sidebands 11 db, all referred to the threshold for 1000 c/s of the subject under concern.

With a mean frequency of 1600 c/s and a modulation rate of 200 c/s the residue with a pitch 200 c/s is clearly audible, though it is partly masked by the strong carrier component. Just as in the case of amplitude modulation the application of the modulation voltage immediately produces a low pitch, leaving the original high pitch only as a timbre.

By increasing the mean frequency the residue shows the pitch course, described in section 3.3, but for higher carrier frequencies it becomes rapidly weaker.

At the point 2000 c/s it looses its tonality, yet there remains only a flutter of low, but indeterminate pitch. At still higher frequencies a pitch impression corresponding to the carrier frequency becomes apparent.

For other modulation frequencies the description is the same. The limiting frequencies for tonality are given in Table V.

The average ratio b/a is definitely lower than the values given previously in table IV. It must be noted that here the effect is isolated, whereas in the previous experiments a corresponding phase condition prevails only between successive bursts of the intermittent residue.

Table V (section 6.1)

Limit of f. m. residue (m = 0.9)

Basis frequency b	Carrier frequency a	Ratio $\frac{b}{a}$
100	1000	0,10
150	1800	0,083
200	2000	0,10
250	2300	0,11
300	2700	0,11
400	4000	0,10

6.2. When the modulation index of the frequency modulated signal is increased, the number of contributing components increases nearly proportionally. The residue pitch becomes more pronounced and the masking influence of the carrier component decreases as the spectrum becomes more uniform. The pitch of such a signal is again confirmed to be the same function of the centre frequency as is described in the preceding chapter.

The spectrum obtained is coherent since the separation between adjacent components is exactly equal to the modulation frequency. It is not surprising to find that a condition that yields an inharmonic spectrum does not lead to audible beats as long as the spectrum does not extend to "negative frequencies". *)

In general the residue effect is markedly less clear in frequency-modulation experiments than it is for amplitude modulation with similar spectrum. This suggests that the residue pitch of signals of nearly constant amplitude may show anomalous properties.

An example of an experiment on approximate frequency modulation is found in the paper of Mathes and Miller (section 1.6). It is reported that a three-component signal in which the phase of the carrier component is 90° out of phase with the main phase of the side bands tends to have a pitch equal to twice the modulation frequency.

In this signal the number of maxima per modulation

*) Here again the frequencies are used as algebraic numbers. Compare sections 2.3 and 3.7.

period is two hence the pitch corresponds to the frequency of the maxima of the envelope.

It is interesting to study approximate frequency modulation somewhat further.

6.3. In Fig. 6.3.1-A the principal components of a frequency-modulated signal for small m (section 6.1) are given in a vector diagram. The rotation of the carrier component with an angular velocity ω is represented by a corresponding rotation of the projection axis on which all components are to be projected. Figure 6.3.1-B shows the same condition at a time where the first sidebands are in phase. The property that the residue is independent of the phase of the centre tone (sections 4.3 and 5.3) makes that the initial angle of the projection axis can be chosen deliberately. Hence the condition of Fig. 6.3.1-B is equivalent to that of Fig. 6.3.1-C.

The latter figure shows clearly how the situation can be approximated by a modification of the amplitude-modulation method, namely by a phase change of 90° of the carrier component (compare Mathes and Miller). The simplest way of obtaining gradually changing phase relations is provided by the use of an incoherent two-component basis signal. One can then investigate which phase of the second sidebands produces the most audible residue (Fig. 6.3.1-D).

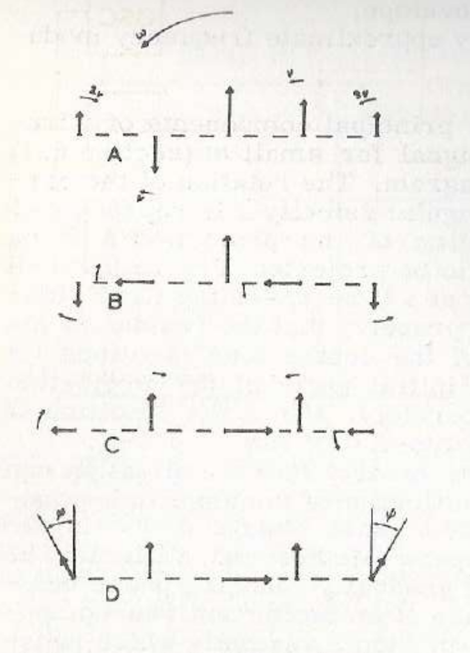
The experimental set-up is shown in Fig. 6.3.2. The carrier phase shift is obtained by using an RC-network fed by a push-pull amplifier. The produced signal can be represented by

$$f(t) = c_0 \sin \omega t + 2 \cos \omega t \left\{ c_1 \cos \mu t + c_2 \cos(2\mu t + \psi) \right\}$$

(compare formula 1 of section 4.3), in which ψ denotes the phase of the highest basis component (see Fig. 6.3.1-D).

The instantaneous value of ψ can be found from the Lissajous pattern produced by the basis components.

The experiment yields a rather weak phase effect. The residue is found to be the most pronounced when ψ is around 90°. This result seems independent of the fre-



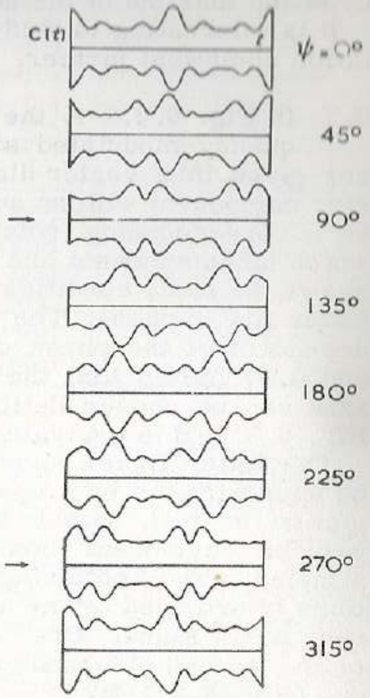


Fig. 6.3.1-A,B,C Equivalent vector representations of frequency-modulated-signal.

Fig. 6.3.1-D: Vector representation of the signal used in the experiment on approximate frequency modulation.

Fig. 6.3.3 Envelope of the signal of Fig. 6.3.1-D for various values of ψ.

quency region of the complex provided the frequencies are not chosen too high. The range of the optimum is not materially influenced by the amplitude distribution of the components when the carrier component contributes audibly to the sound.

The shape of the envelope for various values of ψ is depicted in Fig. 6.3.3. For the calculation (see section 4.2) the values of c_0, c_1 and c_2 are taken equal to 2, 1 and 1 respectively.

It is observed that the experimental optimum around $\psi = 90^\circ$ is present once in a period where ψ changes over 360°. However, similar envelope shapes are present

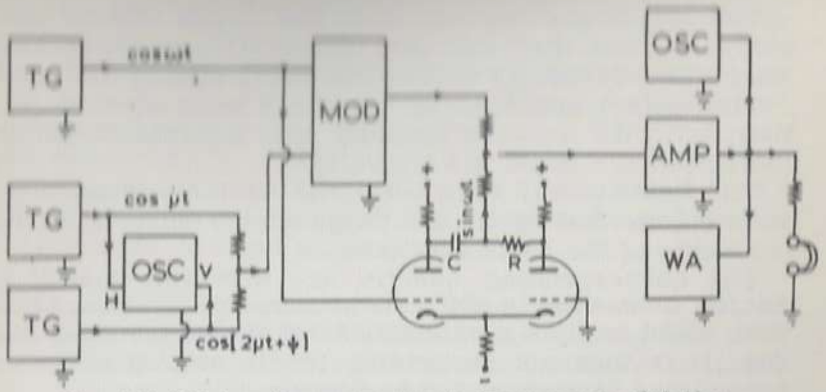


Fig. 6.3.2 Set-up for approximate frequency modulation, TG = tone generator, MOD = modulator, OSC = oscillograph, AMP = amplifier, WA = wave analyser.

twice in this range of ψ. Here we meet a case where the envelope shape does not correspond to the perceived quality.

This deficiency of the envelope concept does not appear unexpected. The envelope has been introduced as a feature that facilitates the separation into pitch and phase effects (sections 4.2 and 4.3). It is not surprising to find that its value is restricted.

In the present case the signal is a hybrid waveform with amplitude as well as frequency modulation.

The signals with $\psi = +90^\circ$ and $\psi = -90^\circ$ differ with respect to the time relation between amplitude and phase modulation. The tone quality thus seems to depend upon this relation.

It is suggested that the pitch of inharmonic signals with both amplitude and frequency modulation might also be dependent upon this relation. This constitutes an interesting subject for future research.

6.4. When a signal with nearly constant amplitude is used, the residue is very weak. The phase effects can only be studied by very attentive listening. In these experiments it is sometimes found that an inharmonic complex produces beats in the residue ("inharmonicity beats").

The frequency of these beats corresponds to the repetition rate given by the deviation of the centre frequency from a harmonic value relative to the basis frequency (see section 3.6).

The beats are weaker than the phase effects under study, so that they are only discernible in experiments where the residue as well as its phase effects are weak.

This effect could not be ascribed to an artifact (section 3.7). We have to conclude that the rule formulated in section 4.3 is not always valid.

The beats occur only when the carrier frequency is rather low, that is in the range of the third to the fifth harmonic of the basis frequency.

The corresponding spectra are that wide that it becomes increasingly difficult to conceive of a mechanism that could explain a smoothness of the inharmonic residue. It is thus not surprising to discover that the rule formulated in section 4.3 breaks down.

6.5. After experiments on a signal with nearly constant amplitude, yet containing a number of equally spaced Fourier components, it is interesting to investigate another extreme of the separation of waveform and spectrum.

The power spectrum of noise is not materially changed by amplitude modulation. The modulation produces certain repetition properties which are reported to be reflected in a pitch-like character of the sound (section 1.7).

In the experiment the modulator is used as an electronic switch. White noise is modulated by a pulse signal of 200 c/s and the carrier is adjusted so as to produce zero signal between the bursts of noise.

The results are to some degree unexpected. Though the pulsed noise can in general be well distinguished from a constant noise, a tonal sensation of the kind aroused by a residue is never perceived.

No observer proved capable of making a reliable pitch match as is possible with a tonal residue of corresponding frequency. When the modulation frequency is below 50 c/s the general harshness of the pulsed noise changes into a flutter. The listener then becomes aware of the temporal structure of the signal. Even so the observers are insensitive to changes in flutter frequency.

It may be expected that a more tonal sound is obtained by filtering the noise previous to modulation. For this purpose a filter is used that provides deliberate control of the frequencies and dampings of two zeros and two

poles in the complex-frequency plane (see Appendix, section A.6).

Experimentally, no change in the general picture is found up to the point where the bandwidth of the noise becomes smaller than the basis frequency. When the primary bandwidth is a fraction of the basis frequency the sound displays a pitch-like character (though still it remains noisy). In this condition the modulation process has profoundly altered the spectrum.

6.6. It is interesting to investigate whether a pulsed noise signal might influence the perception of a sound that can be expected to have an inherent unstable pitch. A "symmetric signal" consisting of odd harmonics of a frequency of 200 c/s is produced by modulating 1800 c/s with the first and second harmonics of 400 c/s. The two possible residue pitches are, as reported, close to 400 c/s, namely 460 and 350 c/s. The repetition rate, however, is 200 c/s. The addition of a pulsed noise with a repetition rate of 200 c/s might be imagined to force the residue mechanism to determine a pitch of 200 c/s.

A preliminary experiment shows that the residue is easily masked by the pulsed noise. In order to decrease this effect the bandwidth of the noise, prior to the modulation process, should be restricted as much as possible. Renewed investigation shows that the addition of the two signals produces no audible interaction other than masking of the weaker of the two.

The pitch of the residue is not influenced by the presence of the noise signal, neither is the pitch impression of the noise signal enhanced by the residue signal.

6.7. Experiments on binaural interactions are generally considered to give evidence on the preservation of time relations in the auditory system.

Miller and Taylor report binaural beats when coherent noise signals are modulated by two slightly different frequencies and the two signals thus obtained are fed each to one ear.

This experiment has been repeated by using two modulators fed by identical noise signals as carrier. The modulation signals are obtained from two channels of the pulse generator, fed by two slightly different frequencies.

On listening the binaural beats are easily perceived. The listener notes the peculiarity that the beats are not mainly characterized by changes in auditory localization, but rather by a repeated transition between raucousness and smoothness of the total sound.

The greatest harshness of the sound is present when the moments of maximum stimulation of the ears coincide. The sound is perceived as a noise of appreciably greater smoothness when the bursts in the ears alternate.

6.8. The preservation of time successions proved by the experiments of the preceding section may be relevant for residue perception. Experiments on binaural interaction of residues may thus be very interesting.

In order to restrict the number of parameters the sounds have been chosen to consist of only three components. The signal for the left ear is produced by modulating the centre tone of frequency a_1 with the basis frequency b_1, whereas a_2 and b_2 are the corresponding parameters of the signal fed to the right ear. The sounds are made of similar waveform and are adjusted to the same loudness.

The parameters a_1 and b_1 were chosen first equal to 1000 and 200 c/s respectively.

1. The basis frequencies are equal. When the centre frequencies are some 4 c/s different, the observer experiences binaural beats having a frequency equal to the difference of the centre frequencies.

By increasing the centre-frequency difference the beats can be followed to a rate of the order of 20 c/s. In this region the pitches of the two signals are estimated to be quite different, yet the beating sensation is quite perceptible. We propose to call these beats "centre beats".

2. The centre frequencies are chosen equal, the basis frequencies differ slightly. Beats are again audible but less evident than in experiment 1. Their frequency is the difference of the basis frequencies. These beats will be called "basis beats". When the listener is asked to change the frequency b_2 in order to make the beats vanish, this proves to be very difficult.

3. The experiment 2 is repeated with markedly different centre frequencies. This difference is chosen 30 c/s so that the "centre beats" degenerate into a flutter. The basis frequencies are chosen slightly different.

It proves to be very diffucult to concentrate upon the "basis beats" in this case. The ability of a test person

for correction of the basis-frequency difference is extremely poor.

4. In this experiment the conditions are reversed. The basis frequency difference is chosen at about 30 c/s, so that the "basis beats" are weakly audible. A small difference in centre frequency is easily perceived. The effect is not as distinct as in experiment 1. The experiment is repeated with the aim to investigate the point where the pitches of the two signals are judged equal. It turns out that on binaural listening the pitch judgements are difficult to make.

This is due to the combined influence of the basis and centre beats which gives the total sound a fluttering character.

It is almost certain that no peculiar effect is experienced at the setting which makes the two tones of equal pitch.

5. The experiment on centre beats is repeated with the initial centre frequencies chosen equal to adjacent harmonics of the basis frequency. The parameter a_2 is chosen 1200 so that the pitches of the sounds are equal. An additional increment of a_2 is again perceptible by the production of binaural beats (centre beats).

6. By chosing $a_2 = 1200$ and introducing a slight difference between b_1 and b_2 the basis beats were audible to a degree comparable to that of experiment 2. From experiments 2, 3 and 6 we conclude that the ability for hearing basis beats is restricted to the case where the initial signals have a few components in common.

7. The experiments 1 to 6 inclusive are repeated with inharmonic signals in the same range. The results are found completely identical, hence it is stated that centre and basis beats are not restricted to conditions where one of the initial signals is harmonic.

Now we want to study other ranges of the parameters a_1 and a_2.

8. The experiments 1 to 7 are repeated in the range where the residue is degenerated. By chosing a value of 2400 c/s for a_1 one observes that the beating sensations are confined to the high tones that can be heard in the signal.

In fact the abstraction of the high tones from the vague flutter of the residue in this condition is considerably facilitated by these beats.

This result makes it highly probable that the beats of the residue in the cases 1 to 7 are caused by loudness and localization changes in the components. The possibility of binaural beats of the high components involved in experiments 8 may well be due to the presented stimuli being complex rather than pure tones.

In experiment 1 all components of the signal fed to one ear were displaced the same amount so that binaural beats are considerably facilitated.

The result that centre beats were more easily perceived than basis beats is clearly in favour of the above assumption.

9. As a final check binaural interaction of harmonic signals is investigated. The two signals consist each of three components and are produced by modulating the centre tone with the lowest component of the output signal of the frequency divider. The generation of completely independent signals is made possible by building the frequency divider in duplicate.

In this case beats corresponding to the centre beats as well as the basis beats can be heard. For small differences in centre frequency the frequency of the beats corresponds to the centre frequency difference.

When this difference is above 4 c/s the centre beats become progressively weaker. At still higher values beats with a frequency equal to the difference of the basis frequencies (basis beats) become apparent.

In fact these beats are far more pronounced than the centre beats appearing for small centre-frequency differences. This is just opposite to the results of experiments 1 and 2 where the centre beats are found more pronounced.

The discrepancy is clearly in favour of the explanation that the centre beats are due to binaural interaction of corresponding components. In experiment 1 a frequency difference equal to the centre-frequency difference is found between each pair of corresponding components. In experiment 9 this frequency difference is only found between the centre components of the signal.

The basis beats are probably due to comparison in time of events evoked by the envelopes of both signals (compare the "precedence effect", see Haas, H-1). In this respect they are related to the binaural beats found for pulsed noise.

When this explanation applies they will become more pronounced when the number of components is increased. The experiments 2 and 9 have been repeated with five and more components and the expected effect is confirmed. We may thus postulate that stimulation patterns according to the envelope of the signals play a role in residue perception.

Chapter VII

DISCUSSION II

7.1. In this chapter the results of various authors will be compared and the remaining discrepancies explained as much as possible. The most important divergence is that between the negative evidence obtained by Fletcher and by Hoogland on the one side and the positive evidence of Schouten and of others on the other side.

Fletcher's experiments were performed at a time when the problem of aural distortion was in the focus of attention and it is not surprising that he adopted the hypothesis that the effect of the missing fundamental is due to the re-introduction of the fundamental by non-linear distortion. In the light of our results we note that Fletcher used a tonal range that is not favourable for an easily detectable residue. First, the order of presented harmonics is small and second, a fundamental frequency as low as 100 c/s gives a residue of reduced tonality.

It is suggested that the dominating position of the common difference tone has obscured the rather faint and elusive character of the residue in this case. The statement quoted in section 1.1 indicates that Fletcher himself was not fully convinced of his distortion hypothesis either.

7.2. Hoogland, who obtained his negative evidence after the residue theory was founded, tried to explain the results of Schouten's by various assumptions regarding deficiencies of Schouten's apparatus. Hoogland's explanation of Schouten's finding that a residue and a pure tone do not interact with each other is not convincing. Hoogland supposed that this effect was due to irregular variations of the angular velocity of the slotted disk of Schouten's optical siren. He stated that the difference tone, showing corresponding variations of frequency, could thus not be detected by a wave analyser or by the search-tone technique.

This inference can be disproved. Two tones, one of which has an inconstant frequency, show a varying phase relation, so that the amplitude of the combination varies as well. The beats in the search-tone experiment will be irregular but just as pronounced as for tones with constant frequency.

In addition it is difficult to see how the anomalous behaviour of interaction could be restricted to the difference tone only. The fundamental, when added to the complex, will show similar frequency variations, while it is reported by Schouten to behave normally in every respect.

Irregular frequency was introduced in our experiments deliberately (section 3.1), and it proved to cause severe impairment of the residue tonality. Since Schouten's residue was highly tonal, the frequency of his signal must have been sufficiently constant.

7.3. The explanation of Hoogland's negative results as regards the residue is to a great extent given by Licklider. He argues (L-3, page 16) that from the research of Mathes and Miller the conclusion must be drawn that the residue is much enhanced by the appearance of peaks and valleys in the waveform.

In Hoogland's experiments the relative phases of the components, though nearly constant, could not be chosen at will. The experimenter had to accept the conditions as they presented themselves. Hence the phase conditions necessary for production of an accentuated waveform will in general not prevail.

According to Licklider, "it appears likely .... that Hoogland missed the residue because he did not control the phases of his five sinusoids in such a way as to approximate an impulsive waveform".

The effect of changing the phase relations is investigated by Licklider in a direct way (section 1.8), and the results fall in line with his suggestions (L-3, page 17; L-4).

Our experiments confirm the phase effects found by Licklider (see section 3.6). In conditions corresponding to Hoogland's the residue is only rarely present, namely when the peaks of the total waveform are very pronounced (extreme case of the intermittent residue, see section 5.8). We can thus suggest that Hoogland's negative result is largely due to the wide separation of complex region and difference frequency. One can suppose that Hoogland's results would have been different when a lower frequency region had been chosen. The following experiment has been designed in order to explore the behaviour of the residue in various regions of the mean frequency.

An incoherent complex is produced by shifting with the double modulator the frequencies of the sum of four voltages produced by four independent tone generators. The original frequencies are 200, 400, 600, 800 c/s. At these low frequencies the tone generators are reasonably stable. It is found that the transition of continuous to intermittent residue when the frequency shift is varied is less sharp than in the case of partially coherent complexes obtained by the modulation method (section 5. 8). Nevertheless, a residue is manifest when the frequency region of a completely incoherent complex is chosen rather low.

This illustrates that, whenever criticism is to be formulated, one had better not disregard the exact conditions used by the criticized author.

7.4. In the research described in this thesis most of the results obtained by Schouten are confirmed. Also his theoretical remarks do apply to a great extent. Before entering the discussion of the remaining discrepancies we need to stress one fundamental point. Schouten described the residue as a subjective component that is not subject to further aural resolution. In fact this definition of the residue implies a kind of recipe for finding it.

In our experiments on phase effects the close relation of the residue to a lack of resolution is again demonstrated. This justifies our adoption of the name „residue" for the phenomenon under concern.

The main subject of this section, however, is a discussion of the discrepancies between Schouten's results and the findings reported in this thesis. The first of these is Schouten's observation that the residue as well as its pitch are most prominent when the complex contains the higher components.

In our experiments we duplicate the finding of Small (S-7) that the tonality decreases with increasing mean frequency. The difference is easily accounted for when one realizes that in Schouten's research the number of components increased with increasing mean frequency.

The existence of a tonal residue arising from a large

number of components with small relative separation is proven by Rosenblith (R-2).

This experiment is duplicated by Licklider (L-3) for demonstration purposes. A series of equidistant pulses is filtered so that only the components above 4000 c/s are passed. The pulse repetition rate is varied from 100 to 1000 c/s. The sounds thus obtained are alternated with sinusoids having the same frequency as the preceding pulse series.

It is demonstrated that the residue pitch is equal to that of the following pure tone and that both show the same musical intervals.

A divergence that is more difficult to explain away bears on the crucial experiment for Schouten's theoretical discussion. He found a signal consisting of pulses of alternating signs to have a pitch equal to the repetition period even when the lower components were suppressed.

In our experiments a corresponding signal always had one of two pitches both deviating somewhat from the difference frequency (section 3. 3).

In order to investigate the discrepancy our experiment was extended to the case of a wide spectrum not unlike that used by Schouten. No tendency for perception of a pitch corresponding to the repetition rate was noticed, however.

The discrepancy was further investigated by duplicating Schouten's experiment as closely as possible.

This is done by synthetizing directly the signal consisting of alternating pulses. The fundamental is cancelled by the addition of a sinusoid of the same frequency. When the production of additional components is carefully avoided, a pitch corresponding to the repetition period is never perceived. This result is not altered when the signal is filtered by the continuously adjustable filter so that only a part of the components is passed.

It is suggested that Schouten's result may have been due to a slight asymmetry in the waveform is his signal.

7.5. From our experiments it is evident that tonality of pulsed noise is nearly absent. This diverges from the results of other investigators.

Miller and Taylor report that the frequency of pulsed noise can, though poorly, be matched to a pure tone. From this they conclude that pulsed noise has pitch. The difference limen for pulse frequency is surprisingly small. These figures are obtained by objective, statistical methods.

However, the criterion for discrimination need not be pitch. The method of subjective abstraction used in our experiments, leads us to suppose that the discrimination criterion is psychologically not related to pitch.

Small (S-7) gives the results of pitch matches of pure tones to complex signals. Results for pulsed noise and for pulsed sinusoids were obtained in a similar way and reported together. Though the matching ability is generally poorer in the case of pulsed noise, the difference is far less than that indicated by our experiments.

The difference may be attributed to two factors. First, Small used comparison to pure tones exclusively. The comparison of two sounds without timbre resemblance seems to impair the accuracy of any judgement, so that differences between various conditions, though observable, are reduced.

Second, Small pooled the results of poor observers in his computations. This too diminished the quantitative expression of the difference found.

That the per cent accuracy for pulsed noise is of the same order of magnitude as that for a pulsed tone is stated not to imply any similarity of subjective tonality (private communication).

Our negative result about the tonality of pulsed noise has recently been confirmed in a paper of Mowbray, Gebhard and Byham (MGB-1).

7.6. The use of inharmonic signals, indicated by Schouten's observations, has in our research led to most interesting results. In various papers the use of such signals is reported.

Fletcher describes that a frequency shift of 30 c/s to a complex with 100 c/s frequency difference destroys its tonality. From our experiments it is evident that it takes a good deal of training to observe pitch shifts in complexes of corresponding parameters. What is pointed out in section 7.1 applies even more for inharmonic signals.

Mathes and Miller report that the presence of a basic pitch is not restricted to harmonic situations. They do not mention, however, whether the pitch of an inharmonic signal deviates from the modulation frequency.

In his paper on pitch effects of amplitude modulated signals Small reports that „the ease with which the periodicity pitch is heard is in no way dependent upon having components of the pulsed tone at frequencies corresponding to harmonics of the repetition rate" (reference S-7, page 757). He further states that the pitch level of inharmonic signals appears exactly to follow the repetition rate when it is swept over a wide range, and the carrier is kept constant.

At first sight the second statement seems to be in contradiction with our results.

Small's figures reveal that he used carrier frequencies higher than the tenth harmonic of the modulation frequency. Under comparable conditions our pitch experiments suffer from decreased accuracy. Pitch shifts due to inharmonicity could even no longer be determined.

For constant centre frequency a change of basis frequency causes a corresponding variation of pitch impression. In this light Small's results are in agreement with our experiments.

7.7 It has often been tried to trace the "Fourier frequency" of a complex sound as distinct from the subjective pitch. In most cases a subject was asked to compare the complex sound with an adjustable pure tone. The intention was to determine if the subject experienced a subjective tone corresponding to the missing fundamental of the complex sound.

Sometimes a preference was indicated for matching the pure tone to the frequency region of the components presented. The conclusion was drawn that for such sounds two pitches are perceived, one corresponding to the repetition frequency, the other to "Fourier frequency" (S-7, DSM-1, L-2).

Once it is realized that this is incorrectly expressed, the inherent limitations of the method become apparent. For the phenomenon under concern the purely subjective method of Schouten's is to be preferred. Only when the fundamental evidence is clear to the experimenter he may design psychophysical methods for evaluation of quantitative data.

These should involve the matching of not too far different sounds.

The impossibility of even crude determination of Fourier frequency of a complex yielding a residue is obvious. The frequency region of the complex is perceived as a vague timbre.

The only way of determining it is comparison with narrow band noise of similar bandwidth and adjustable frequency region. Even then it can be doubted if anything consistent is borne out, in view of the profound variations of timbre with the phase relations.

Chapter VIII

THEORETICAL ASPECTS

8.1. The signals used in the experiments described show certain regularities in the waveforms as well as in the Fourier spectra. These constitute two aspects of the signals that may both be relevant in determining pitch.

Waveform and spectrum properties will in this chapter be worked out separately so as to give rise to two basic schemes of the determination of the residue pitch. These will be called waveform and spectrum descriptions respectively.

For harmonic signals the residue pitch corresponds to the repetition rate of the waveform and also to the greatest common divisor of the component frequencies.

We will try to find extensions of the concepts of period and fundamental frequency that apply to inharmonic signals as well. These extensions will be called pseudo-period and pseudo-frequency.

It is supposed that the residue pitch is determined by the ear from either of these aspects of the signal. The expected pitch of an inharmonic signal can then be compared with the experimental data given in section 5.2. When the agreement is reasonable a tentative residue theory can be proposed that has its basis analogue in either the waveform or the spectrum description. By taking into account the physiology of the ear as far as it is known to-day, a choice between the two possibilities may be attempted.

8.2. The period of a harmonic signal is the smallest time separation after which the waveform repeats itself. For a slightly inharmonic signal there is still a similarity between the waveforms in successive periods of the envelope. This leads to the concept of pseudo-period.

Consider a harmonic complex, the centre tone of which is the k^{th} harmonic of the basis frequency b. The period is $1/b$. The complex is made inharmonic by a centre tone deviation d. For small d one can determine a time separation over which the similarity of the waveforms in successive periods of the envelope is maximal. This "pseudo-period" is indicated by the arrow in Fig. 8.2.1-A. It is a continuous function of d and, consequently, of α_k (which is equal to d/b).

It becomes equal to the true periodicity $1/b$ when α_k is zero.

In general there is an ambiguity in the choice of the pseudo-period. This is easily demonstrated when α_k is increased beyond $\frac{1}{2}$. The pseudo-period can be extended continuously for these values. As Fig. 8.2.1-B shows, the pseudo-period still represents a time separation of similar waveforms though the similarity is no longer optimal.

For a given signal several pseudo-periods can be found that may differ in degree of similarity (Fig. 8.2.1-C).

This ambiguity parallels the multiplicity of the inharmonicity index (section 2.5). The pseudo-periods can be distinguished by attaching the subscript j so that the pseudo-period P_j becomes equal to $1/b$ when the corresponding α_j is zero. One of these indicates maximal similarity. This is called the principal pseudo-period. In view of the definitions given in section 2.5, it will be denoted by P_k.

Notwithstanding the crude definition of pseudo-period given above, a good estimate of the dependence on centre and basis frequency can be obtained. When the envelope is a slowly varying function of time, the pseudo-periods are nearly completely determined by the fine-structure of the waveform, which in its turn is given by the centre tone a. All pseudo-periods will thus be nearly independent of the basis frequency. For a harmonic complex, the centre tone of which is the k^{th} harmonic of the basis tone, the parameter α_k is zero. Hence the principal pseudo-period P_k is equal to the basis period. It is then reasonable to assume that the pitch of an inharmonic complex with small α_k corresponds to P_k (waveform description).

Fig. 8.2.1-C reveals that P_j is nearly equal to j times the centre-tone period. This applies to P_k as well. The expected pitch for small α_k will thus be equal to a/k. This corresponds to the "first approximation" of the residue pitch (section 3.4).

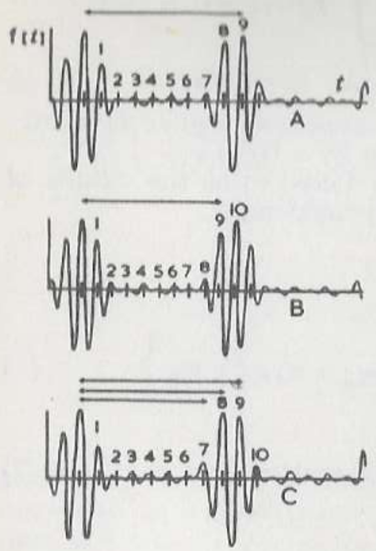


Fig. 8.2.1 Waveforms and pseudo-periods of inharmonic signals.
A: α_9 = -0,4; the arrow indicates P_9
B: α_9 = +0,8; the arrow indicates P_9
C: α_8 = +0,4; the arrows indicate P_7, P_8, and P_9 in order of increasing length.

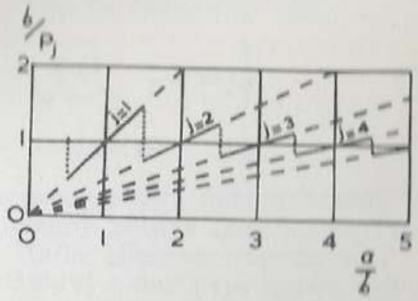


Fig. 8.2.2 Expected pitch as derived from the pseudo-period.

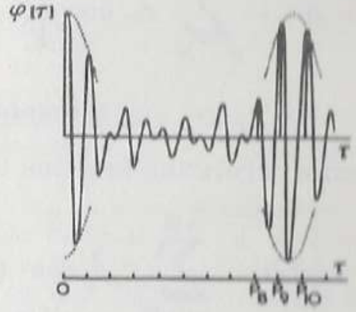


Fig. 8.3.1 Typical example of $\varphi(\tau)$. The dotted lines indicate the envelope $C(\tau)$.

Fig. 8.2.2 shows the expected pitch for various values of a/b. The full lines indicate pitches derived from principal pseudo-periods. The dashed lines are continuous extensions, which correspond to the other pseudo-periods.

8.3. A more accurate definition of pseudo-period can be provided by the concept of autocorrelation.

When the signal is represented by $f(t)$, the autocorrelation function $\varphi(\tau)$ is defined as

$$\varphi(\tau) = \tfrac{1}{2} \lim_{T \to \infty} \frac{1}{T} \int_{-T}^{+T} f(t-\tau)\, f(t)\, dt$$

This function represents an average degree of similarity of signal values separated by a time τ.

The pseudo-periods shall be taken to be the values of for which $\varphi(\tau)$ has a (relative) maximum.

For the signal

$$f(t) = \sum_{n=-N}^{+N} c_n \cos \left\{ (k + \alpha_k + n)\, \mu\, t + \psi_n \right\} \qquad (1)$$

(compare equation 1 of section 4.2)

the autocorrelation function is

$$\varphi(\tau) = \sum_{n=-N}^{+N} c_n^{\,2} \cos\ (k + \alpha_k + n)\, \mu\, \tau \qquad (2)$$

The phases ψ_n of the components of (1) do not appear in (2) since according to the Wiener-Khintchine theorem the autocorrelation function is the Fourier transform of the power spectrum of the signal.

The generalized amplitude (see section 4.2) of $\varphi(\tau)$ is given by the complex function

$$c(\tau) = \sum_{n=-N}^{+N} c_n^{\,2} \exp\ i\, n\, \mu\, \tau .$$

When $c(\tau)$ is written as

$$c(\tau) = C(\tau) \exp\ i\ \psi(\tau),$$

the autocorrelation function can be expressed by

$$\varphi(\tau) = C(\tau) \cos \left\{ (k + \alpha_k)\mu\tau + \psi(\tau) \right\} \qquad (3)$$

in which the envelope $C(\tau)$ of $\varphi(\tau)$ is given by

$$C(\tau) = \sqrt{(\Sigma\, c_n^{\,2} \cos\ n\mu\tau)^2 + (\Sigma\, c_n^{\,2} \sin\ n\mu\tau)^2}$$

(compare section 4.2)

The envelope has a maximum at

$$\tau = \frac{1}{b} = \tau_0$$

and is symmetric with respect to this point. Since the envelope is periodic the curvature at $\tau = \tau_0$ is equal to that at $\tau = 0$.

Hence the curvature at $\tau = \tau_0$ is always negative. The phase-modulation term $\psi(\tau)$ is zero at $\tau = \tau_0$. For a signal with symmetric spectrum ($c_{-n} = c_{+n}$) it is zero for any value of τ. An example of the autocorrelation function in this case is depicted in fig. 8.3.1.

The principal pseudo-period is the value of τ for which $\varphi(\tau)$ has an absolute maximum in the range around τ_0. When α_k is zero this maximum coincides with that of the envelope at τ_0. For small α_k the value of P_k is close to τ_0.

In order to obtain a crude estimate of P_k we introduce two approximations. First, the phase modulation $\psi(\tau)$ is taken zero. This is fully legitimate in the case of a symmetric spectrum. Second, the envelope $C(\tau)$ is taken as constant in the range around τ_0. It is now easily seen that P_k is found by putting the argument of (3) equal to 2π k:

$$(k + \alpha_k)\, \mu\, P_k = 2\pi k \qquad (4)$$

Thus P_k is equal to k times the centre-tone period. The expected pitch level is then a/k. This corresponds to the "first approximation" introduced in section 3.4.

A more accurate solution can be obtained by taking into account the curvature of $C(\tau)$ in the range of τ around τ_0. This curvature is negative so that the maxima of $\varphi(\tau)$ are displaced from the value given by (4) in the direction of τ_0. The expected pitch shifts due to inharmonicity are then smaller than those predicted by using the "first approximation" concept.

Experimentally it is found that the pitch shifts are larger than the "first approximation" values.

The agreement between experimental and predicted values can be improved by taking into account the phase modulation of $\varphi(\tau)$.

As said, $\psi(\tau)$ is zero when the spectrum is symmetric with respect to the centre tone. The experiments have been performed with symmetric objective spectra only. Phase modulation of the autocorrelation function can be important only when the symmetry is disturbed by filtering of the signal before the analysis is made.

The coefficients c_n of $\varphi(\tau)$ now include some weighting factor assigned to the components.

In order to improve the agreement the weighting factors have to be a decreasing function of frequency (compare section 5.4).

8.4. In this section the concept of fundamental frequency will be extended toward inharmonic complexes. The derivation consists essentially in looking for a frequency near to the basis frequency that is an approximate common divisor of the presented frequencies. This frequency is called pseudo-frequency.

The component frequencies are represented by

$$(k + \alpha_k + n)\, b \qquad (1)$$

$$(n = -N,\ -N+1,\ \ldots\ +N-1,\ +N).$$

The frequency p_k for which the harmonics

$$(k + n)\, p_k \qquad (2)$$

approximate the given frequencies as much as possible is the pseudo-frequency. In the spectrum description we assume that the pitch is equal to the pseudo-frequency.

As a quantitative measure of the accuracy attained we determine the sum S of the squared differences of (1) and (2), each term containing a positive weighting factor w_n which includes the amplitude of the corresponding component.

The sum S is now given by

$$S = \sum_{n=-N}^{+N} w_n \left\{ (k + n + \alpha_k) b - (k + n)\, p_k \right\}^2$$

The given frequencies (1) are approximated by the harmonic series (2) with maximal accuracy when S is minimum (method of least squares).

Solving the equation

$$\frac{d\,S}{d\,p_k} = 0$$

for p_k yields

$$p_k = b + \frac{b\,\alpha_k}{k}\; \frac{\displaystyle\sum_{n=-N}^{+N} w_n \left(1 + \frac{n}{k}\right)}{\displaystyle\sum_{n=-N}^{+N} w_n \left(1 + \frac{n}{k}\right)^2}$$

The second term represents the expected pitch shift due to inharmonicity. The second factor of this term is in most cases nearly equal to unity.

The formula can be simplified by the introduction of the effective k. This is defined by

$$\frac{k}{k_{eff}} = \frac{\displaystyle\sum_{n=-N}^{+N} w_n \left(1 + \frac{n}{k}\right)}{\displaystyle\sum_{n=-N}^{+N} w_n \left(1 + \frac{n}{k}\right)^2} \quad \text{(compare section 5.4)}$$

Substitution into (3) yields

$$p_k = b + \frac{b\,\alpha_k}{k_{eff}} \tag{4}$$

When k_{eff} is taken equal to k, as is admissible in many cases, this reduces to

$$p_k = \frac{a}{k} \tag{5}$$

This is a pseudo-frequency corresponding to the approximate principal pseudo-period given by equation (4) of section 8.3.

This result does not appear unexpected. Due to the relation between power spectrum and autocorrelation function the derivations of this and the preceding sections are essentially similar. A separate treatment is given here as a basic analogue of a possible hearing mechanism.

When the summations in (3) are carried out under the assumption that $w_{-n} = w_{+n}$, k_{eff} is found higher than k. This means that the expected pitch shifts due to inharmonicity are smaller than those predicted by (5). The agreement between pseudo-frequency and the experimental residue pitch can be improved only when w_n is a decreasing function of frequency, such that k_{eff} becomes smaller than k.

Since the dependence of pitch upon the complex width is small, the effect of the presence of one particular component is by far too small to be measured. The weighting factors thus cannot be determined directly.

In the present derivation, w_n represents the subjective importance of a component. Although the experimental results bear on the case of equal objective amplitudes of the components, the subjective amplitudes may not be equal. Furthermore, it seems reasonable to assume that frequency differences are measured by the ear according to a scale which has the difference limen as unit. In this way a physical picture of the weighting factors can be obtained.

In order to show the effect of various substitutions we take w_n as

$$w_n = \left\{ \frac{k}{k+n} \right\}^z .$$

The table shows for various values of z the correction factor k/k_{eff} in cases corresponding to the experimental conditions. For comparison the corresponding data of the ratio of measured pitch shifts and "first approximation" values is included.

Table of k/k_{eff} (section 8.4)

N	k	k/k_{eff} exp.	theor. z=0	z=1	z=2	z=3
2	5	1,15 *)	0,93	1	1,09	1,19
2	8	1,25 *)	0,97	1	1,03	1,07
2	12	1,17 *)	0,99	1	1,01	1,03
2	9	1,15 **)	0,98	1	1,02	1,05
4	9	1,24 **)	0,92	1	1,10	1,21
7	9	1,76 **)	0,81	1	1,46	2,15

*) average of measurements on basis frequencies of 150, 200, 300 and 400 c/s.

**) basis frequency 200 c/s, see Fig. 5.4.1.

When the difference limen is taken as proportional to frequency the value of the exponent z is two. From the table it is seen that the correction factor is still too small to account for the experimental data.

The experimental values of the correction factor are nearly independent of k when the number of sidebands N is constant. In order to explain this z must increase with k.

8.5. In section 8.3 the principal pseudo-period is calculated as the maximum that is closest to the maximum of the envelope of the autocorrelation function. The other maxima represent the pseudo-periods P_j introduced in section 8.2 with j differing from k. In crude approximation P_j is equal to j times the centre-tone period.

This ambiguity of the pseudo-period is introduced upon the consideration that the problem under concern may not have a unique solution.

A similar plurality of solution emerges from the treatment of section 8.4 when it is realized that the given frequencies

$$(k + \alpha_k + n)\, b \qquad (|\alpha_k| \leqslant \tfrac{1}{2})$$

can be approximated by the harmonic series

$$(k + j)\, p_j \qquad (j \neq k).$$

Solution of p_j yields

$$p_j = b + b\,\frac{k - j + \alpha_k}{j}\;\frac{\sum\limits_{n=-N}^{+N} w_n \left(1 + \frac{n}{j}\right)}{\sum\limits_{n=-N}^{+N} w_n \left(1 + \frac{n}{j}\right)^2}$$

When again the correction factor is taken unity the approximate solution is

$$p_j = \frac{a}{j}.$$

Still other pseudo-frequencies can be obtained when the given frequencies are compared to a series of harmonics which are not adjacent to each other.

The resulting pseudo-frequencies correspond to pseudo-periods appreciably larger than the basis period.

These extended solutions can describe the mutual correspondence between different properties of the residue which are at first sight not related. This is illustrated by the following treatment of inharmonic signals.

Consider an inharmonic signal the centre tone of which is $kb + d$. When d is a submultiple of b, say

$$d = \frac{b}{m} \qquad \text{(m integer)}$$

all components are harmonics of the frequency b/m. The signal has then an exact repetition period m/b which is equal to (mk + 1) times the period of the centre tone. When m is large this period is long, it may be of the order of seconds.

The sound is expected to be perceived as fluctuating according to this period of repetition ("inharmonicity beats", see sections 3.6 and 3.7).

Experimentally, inharmonicity beats are absent when exceptional circumstances (section 6.4) are excluded).

When d is increased from the value b/m one can find an approximate repetition period which is a continuous extension of the period m/b.

This will closely follow (mk + 1) times the centre-tone period.

If d approaches the next submultiple of b:

$$d \rightarrow \frac{b}{m - 1}$$

a new pseudo-period, nearly equal to (mk - k + 1) times the centre-tone period, indicates a better approximation of long-time periodicity.

When d is increased from zero, the submultiples of b are lying close together and the long-time period is on the average equal to $1/d$. As said above, it is found that the auditory sensation generally does not repeat itself with a frequency nearly equal to d.

For still higher values of d the frequency corresponding to this pseudo-period enters the auditory range.

The experiments show that a corresponding pitch sensation does not arise.

Even when $d = \frac{1}{2}b$, where the components become odd multiples of $\frac{1}{2}b$, the pitch certainly does not correspond to the exact repetition rate $\frac{1}{2}b$ (see sections 3.3 and 7.4).

From this derivation it follows that the absence of "inharmonicity beats" is related to the absence of a pitch sensation corresponding to the exact repetition period of a "symmetric signal". One can conclude that the experiments indicate that the pseudo-grequencies p_j and the pseudo-periods P_j with an index appreciably larger than k are not relevant to pitch.

The question why the experimentally found pitch corresponds most closely to the principal solutions p_k and P_k constitutes a separate problem.

In view of the definition, P_k indicates the largest maximum of the autocorrelation function in the range around τ_0. The pseudo-frequency p_k given by equation (3) of section 8.4 leads to a smaller value of the function S than the solutions p_j given by equation (1) of this section, provided that the absolute value of α_k is smaller than $\frac{1}{2}$, and j is of the same order of magnitude as k. Thus p_k indicates the closest approximation of a fundamental frequency.

From this discussion it is understood that, if pitch is determined by the ear in a way analogous to one of the methods treated here, the pitch will correspond most closely to P_k or p_k.

The only occasion where experimental evidence for a pitch corresponding to the other solutions (P_j and p_j with $j \neq k$) might be present is found at the high end of the centre-frequency range (see section 5.2). The accuracy of the experiments is, however, insufficient to disclose whether the judgements actually correspond to these solutions or not.

8.6. The waveform and spectrum descriptions treated in the preceding sections are introduced as two basic schemes of residue theory. The concept of pseudo-period, as an approximate period, may be relevant to a neural mechanism that acts upon a cochlear vibration more or less similar in waveform to the presented signal. The pseudo-frequency as an approximate fundamental frequency could describe the ultimate result of an action elicited by completely resolved components. Since the theoretical results of both types of description are so highly similar, no further light is shed upon the actual mechanism that underlies the residue phenomenon.

We cannot resort to physiological data on the amount of resolution before the residue pitch is determined. We can suppose that this resolution is carried out by nearly linear means such as mechanical filtering. Supposing that the resolution is insufficient for separating the components, a residue theory can be developed according to

a basic pattern given by the waveform description. Wh
the resolution is nearly complete, a theory according
the spectrum description emerges.

Data on the over-all discrimination power of the e
(discrimination of pitch, critical bandwidth for maskir
indicates that the relative discrimination is in crude a
proximation independent of frequency. It seems reaso
able to suppose that the resolution of the ear before t
residue pitch is determined can be described by a co
stant relative bandwidth.

The components of a complex with low ratio k of cent
and basis frequencies are then nearly completely sep
rated. This means that for this case the spectrum d
scription applies.

For complexes with high values of k the relative sep
ration is so small that separation by nearly linear mea
is improbable. These signals can be treated in terms
the waveform description.

8.7. In the experiments it is observed that beats ari
when the spectrum of an inharmonic signal exten
into the region of "negative frequencies" (see secti
3.7). These inharmonicity beats are generally not hea
when the spectrum is confined to positive frequencie
When the number of components is low, this differen
of properties can be demonstrated only when the ratio
of centre and basis frequencies is low. This implies th
the relative separation of the components is large. It
reasonable to suppose that here the resolution of t
components is nearly complete.

The effect is easily explained in terms of the spectru
description, namely by taking into account that comp
nents of nearly equal magnitude of frequency interac
Of course the effect can theoretically be explained
terms of the waveform description as well. It is, how
ever, unreasonable to suppose that the signal, witho
substantial filtering, is subjected to autocorrelation ana
ysis.

For complexes of low k the theory based on the spe
trum description may be a simple extension of the pu
place theory. The residue of a harmonic complex can
explained in terms of the place theory by postulati
neural connections between the places corresponding
harmonically related frequencies. These systematic co
nections can be thought to be organized by the repetitio

excitation by harmonic sounds such as occur in everyday life. This concept of self-organization is not uncommon in modern neurophysiology.

The residue for inharmonic signals can be explained as well, namely by taking into account that the accuracy of the connection scheme is necessarily limited. The pitch will correspond to the frequency the harmonics of which approximate the given tones. The common divisor method describes the result of this comparison.

This explanation of the residue does not contain phase effects, which is in agreement with the experimental results for this case of "low complexes".

8.8. An actual mechanism according to the autocorrelation method may apply in cases where resolution is incomplete, so that a few components interact to give a compound vibration. This may be the case for complexes with high ratio of centre and basis frequencies, where the relative separation of the components is small.

Such a mechanism may very well admit of phase effects accompanying changes of the envelope. The pure autocorrelation method implies precise multiplication and integration. Even when the integration is incomplete (Fano, F-6) phase effects may be irrelevant. Precise multiplication is physiologically unreasonable, so that a physiological mechanism that acts on waveform aspects may explain phase effects.

First we treat the case where a coherent signal is presented with so small a relative separation of the components, that cochlear resolution is far from complete. We assume that an approximate period of an inharmonic signal is determined from the time distances of successive neural pulses. When the neural pulses are synchronized by the signal (Wever's frequency principle) most of the pulses will be separated by a time distance equal to several periods of the fine-structure (section 4.2) of the waveform of the stimulus.

When the envelope of the stimulis shows pronounced peaks, a time separation close to the envelope period will often appear. For a complex of which α_k is small, the most probable group of time-distances consists of separations equal to k times the centre-tone period, that is, to the principal pseudo-period. It is assumed that this is detected by the mechanism (pattern-recognition) and converted into pitch. The pitch level cannot be other

than inversely proportional to this time separation because the theory has to apply to pure-tone as well as complex stimuli. The pitch is then found close to the "first approximation" value.

For an inharmonic complex the pulses will not be equidistant, since the instantaneous phase of the fine-structure with respect to the envelope varies continually. The concept of pattern-recognition, proposing that the most numerable time distances are utilized, is introduced in order to explain the stability of pitch for an inharmonic complex. Due to the natural lack of synchronization and to inconstant synaptic delay, a similar irregularity will appear when a harmonic signal is presented. From this we infer that a process of averaging must be involved as well.

The task of the mechanism is considerably simplified by the appearance of peaks in the envelope. When on the contrary the amplitude is constant, the neural pulses will be nearly random with an average distance of about the centre-tone period. This may give rise to the perception of an impure pitch corresponding to the centre frequency. In this way, the extreme phase effects of a complex with small ratio of basis to centre frequency are understood.

A signal the envelope of which has two or more peaks per basis period of about equal height is expected to have a pitch close to a harmonic of the basis frequency. In our experiments on approximate frequency modulation a corresponding trend has been observed when the relative bandwidth is small (also compare Mathes and Miller, MM-1).

8.9. For signals with larger relative bandwidths the cochlear filtering process takes part in the mechanism. At a certain place of the cochlea only a few components interact. The envelope of the stimulus will have a period equal to the basis period when at least three components contribute to the stimulation.

The most probable distance of neural pulses coming from one place will now be an integral number of periods of the middle component of the interacting components (appropriate centre component). It still will be close to the basis period.

We assume now that pitch is determined from a weighted average of these approximate periods over the places of maximal interaction. Pitch is now determined in a way between the methods given in the waveform and spectrum descriptions. The weighting factors introduced into the calculations (section 8.3, 8.4 and 8.5) have now a different meaning. They include the extensions of the regions of maximal interaction as well as the amplitudes of the components and the physiologic scales on which the neural patterns are projected.

By this theory, phase effects in cases of intermediate values of k can be explained as well. Let us assume that the objective signal is transmitted to the appropriate cochlear region with constant group velocity. When the phases ψ_n (see section 4.2) of the objective signal are zero, the envelopes of the stimulating signals at the places of maximal interaction are in phase. This suggests that in this case the action is simplified, so that phase effects can be expected. The phase effects will still be related to the envelope of the objective signal.

This case merges into one that corresponds to the spectrum description when the resolution of the components approaches perfection. When Wever's volley principle applies, the neural pulses coming from a place stimulated by but one component will be synchronized. There will no longer exist a group of time distances nearly equal to the basis frequency. When pulses coming from different regions are compared by way of approximate coincidences, it is possible to obtain a series of pulses where time distances corresponding to the residue pitch outnumber the other separations. A process of pattern recognition similar to that proposed in section 8.8 may explain the residue pitch.

This case corresponds to that treated in section 8.7 except that now the frequency is preserved in the neural pulses (Wever's "frequency principle"). This concept has various advantages. First, pitch bears on similar aspects for pure tones and for residues. Second, the function of integral numbers in hearing is evident from the physical system and need not be acquired by repetitious auditory experience.

8.10. In sections 8.8. and 8.9 the simplest possible description that explains all basic facts is given. The three cases treated, narrow, medium, and wide-band phenomena, did not show large differences in expected properties. Yet we have to discuss the bandwidth of the mechanical analysis of the cochlea.

The bandwidth found by von Békésy is commonly considered too wide to be relevant to direct explanation of aural discrimination. Improved resolution can be obtained by assuming that actual stimulation involves the higher derivatives of the vibration pattern. This implies a great accuracy and in addition, the process is susceptible to interference by noise of physiological origin. Even mechanical sharpening up to a point where the mechanical bandwidth becomes comparable to the critical bandwidth, seems to be hardly possible.

It is therefore reasonable to assume that the stimulation pattern of a pure tone has a width corresponding to several critical bandwidths. The resulting pattern is in its turn analysed by the lower neural parts of the system. There is ample anatomical basis for this assumption (Ranke, R-3). It seems almost inevitable to assume that the sharpening process utilizes repetition properties of the signal, or in other words that neural pulses are synchronized at least at this intermediate stage. It is not possible to state explicitly whether the information about pitch is ultimately transmitted to the brain as a sequence of nerve discharges or as a specific pattern of neural activity.

We turn now to the case of a coherent complex. When the bandwidth of the signal is comparable to the mechanical bandwidth of the cochlea, the stimulation patterns overlap a good deal. It follows that the neural mechanism for perception of single pure tones is impossible. The perception of a residue may now happen in a way crudely described in section 8.8. Unless we specify the actions elicited by a pure tone, we cannot be precise in the description of residue perception. It is, however, evident that the residue phenomenon is closely connected with insufficient mechanical analysis. For a "high" complex with nearly constant amplitude something analogous to pure-tone perception remains possible as is evident from our results.

When the basis frequency is increased, the overlapping of the stimulation areas of the components is diminished. Such a change may lead to the perception of a (continuous) residue for all phase relations. When the basis frequency is higher than the mechanical bandwidth of the cochlea, the stimulation patterns do no longer overlap, and the situation is in favour of determination of residue pitch according to the common divisor method. It is highly possible that a change of properties is involved in the latter transition. This might explain the gradual disappearance of phase effects (sections 5.5 to 5.7 inclusive) and the possibility of "inharmonicity beats" (section 6.4).

One can then conclude that the relative mechanical bandwidth may correspond to the critical ratios given in Tables I, II and III (Chapter V). Such a mechanical bandwidth is throughout reasonable.

This composite picture seems at the moment the most promising description of the residue phenomenon. We should like to emphasize that the residue pitch is regarded as the result of a process of averaging. This offers the most promising explanation of the experimental data.

The present situation calls for further investigation of physiological processes in the case of complex rather than pure-tone stimuli. Although the actions involved when pure tones are presented are already found to be complicated, this constitutes the only way of obtaining clear insight into what happens prior to auditory experience.

SUMMARY

The research reported in this thesis is devoted to the auditory phenomenon of the residue (defined by Schouten as the combined impression of unresolved components). The tone complexes have been produced mainly by amplitude modulation. The residue of a harmonic signal has a pitch that corresponds to the fundamental frequency.

Particular attention has been paid to the perception of inharmonic signals. An inharmonic signal consisting of equidistant components sounds as smooth as a harmonic one with similar parameters. The pitch level of such signals is determined by matching a harmonic complex with variable fundamental frequency. For constant modulation frequency the pitch is found dependent upon the carrier frequency. The relative pitch shifts due to inharmonicity are slightly larger than the relative variations of the carrier frequency. When pitch is plotted versus carrier frequency one finds a saw-tooth curve of which each segment is centered around a harmonic situation.

Inharmonic complexes whose frequencies are not equidistant show beats which correspond to the continual phase variations involved. These phase effects are related to changes in the envelope of the waveform. This correspondence is found to apply in all cases studied except for conditions of approximate frequency modulation.

Complexes with a wide relative separation between adjacent components exhibit somewhat different properties. Such signals show small or negligible susceptibility to phase changes and, in addition, the inharmonic complexes tend to produce beats related with the deviations from harmonicity.

The results of other authors in the field of subjective tones are discussed and most of the divergences explained.

In a theoretical discussion it is concluded that the most probable theory of the residue postulates that approximate periods in the signal are the basis for pitch determination. The fundamental properties of inharmonic

signals as well as the deviations of pitch from the expected values are attributed to a weighed average of the information coming from partially resolved vibrations. The phase effects are explained in part by the facilitation of this process when special phase relations prevail.

For spectra of widely separated components a mechanism like the evaluation of an approximate common divisor to the presented frequencies is suggested. Both mechanisms are supposed to complement each other so that no sharp transition is found.

SAMENVATTING

Dit proefschrift beschrijft een onderzoek naar het z.g. „residu", door Schouten gedefinieerd als de gewaarwording van een groep tonen, die door het oor niet gescheiden worden. Voor de productie van de signalen is gebruik gemaakt van amplitude-modulatie. De toonhoogte van het residu van een harmonisch complex komt overeen met de grondtoonfrequentie.

Bijzondere aandacht is geschonken aan de waarneming van anharmonische signalen. Een anharmonisch complex waarvan de componenten een constant frequentieverschil hebben, is qua klankkleur en tonaliteit niet te onderscheiden van een harmonisch complex. De toonhoogte van de gebruikte signalen is gemeten door als vergelijkingsobject een harmonisch complex met instelbare grondtoonfrequentie te gebruiken. Bij constante modulatiefrequentie is de toonhoogte afhankelijk van de draaggolffrequentie. In de nabijheid van een harmonische situatie verandert de toonhoogte iets sterker dan evenredig met de draaggolffrequentie. De kromme van toonhoogte tegen draaggolffrequentie heeft de vorm van een zaagtand waarvan elke opgaande tak rondom een harmonische situatie ligt.

Anharmonische signalen, waarvan de componenten niet equidistant zijn, geven zwevingen te horen die overeenkomen met de voortdurende veranderingen van de fazen der componenten. Tussen deze faze-effecten en de golfvorm van de omhullende van het signaal kan een wederkerige samenhang gevonden worden. Deze is onafhankelijk van de anharmoniciteit van het complex. Slechts in experimenten met signalen die frequentie-gemoduleerde golfvormen benaderen kunnen afwijkingen van deze regel gevonden worden.

Complexen met een relatief grote afstand tussen de componenten onderscheiden zich experimenteel door afwijkende eigenschappen. Het residu is weinig of niet gevoelig voor faze-verhoudingen en anharmonische complexen geven aanleiding tot het waarnemen van zwevingen.

De resultaten van vroegere experimenten op het gebied van het residu zijn critisch beschouwd en, voor zover mogelijk, worden de afwijkingen tussen de verschillende proeven verklaard.

In de theoretische beschouwingen wordt aannemelijk gemaakt, dat de toonhoogte van het residu voor anharmonische signalen bepaald wordt aan de hand van benaderde periodiciteiten die in deze signalen te vinden zijn. Het is dan noodzakelijk een zekere gewogen middeling van de verschillende pseudo-periodiciteiten te veronderstellen, om alle gevonden eigenschappen te verklaren. De afhankelijkheid van de faze kan worden verklaard door aan te nemen, dat de taak van het mechanisme vergemakkelijkt wordt voor signalen met gepiekte golfvorm.

Voor complexen met een relatief grote afstand tussen de componenten is het van voordeel de bepaling van de toonhoogte analoog te veronderstellen aan het vinden van een benaderde grootste gemene deler van de frequenties die weinig afwijkt van de verschilfrequentie. De twee mechanismen vullen elkaar zodanig aan, dat, van „lage" naar „hoge" complexen gaande, slechts gradueel verlopende eigenschappen kunnen worden gevonden.

Appendix

DESCRIPTION OF APPARATUS

A.1. Pulse generator, single modulator, frequency divider, noise generator and continuous filter have a common supply unit. This unit is of the electronically regulated type and delivers voltages of +340 V and -90 V at a maximum current of 300 mA. The "earth" terminal must be regarded as a reference point. The current, either positive or negative, must be limited to 15 mA.

The circuit diagram of the stabilizing part is given in Fig. A.1.1. The input is fed from a conventional rectifier (choke-input type) of 700 V d.c. output. Throughout the stabilization circuit difference amplifiers of the "long-tailed-pair" type are used for comparison of actual and reference voltages and for driving the regulation valves.

A similar unit is used for the supply of the double modulator.

A.2. The pulse generator consists of three nearly identical channels. In the schematic diagram (see Fig. A.2.1) only one channel is shown.

The input signal is amplified by a Schmitt-type phase inverter circuit, so that both polarities are available for triggering the channels of the pulse generator. The second stage consists of a discriminator circuit which converts the trigger signal into an approximate square wave.

The time of the steep rising and falling phases is controlled by the bias setting (potentiometer I) of the discriminator. The output signal is fed to a Schmitt trigger circuit (Elmore and Sands, ES-1). The nearly perfect square wave thus obtained is sometimes used in the experiments with pulsed noise.

When shorter pulses or greater facilities of control of length and amplitude are required the remaining parts of the circuit can be used. The square wave initiates the action of a phantastron-like circuit. The pentode commonly used in this type of circuit is replaced by three

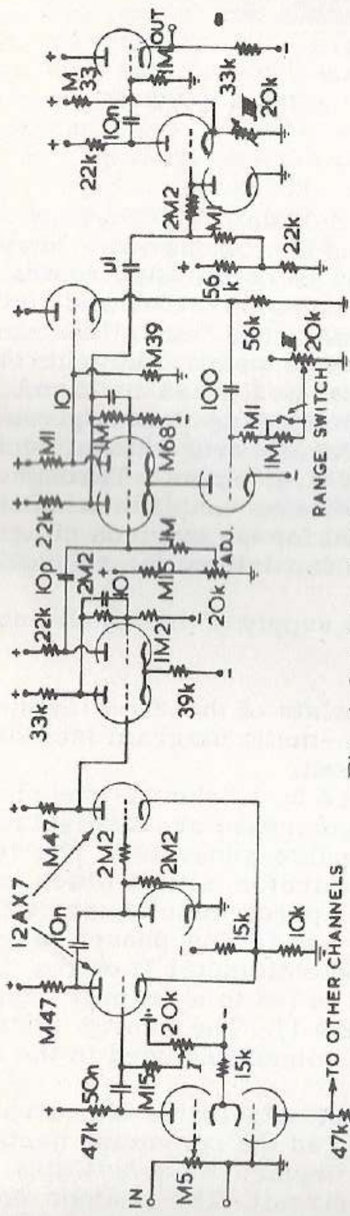
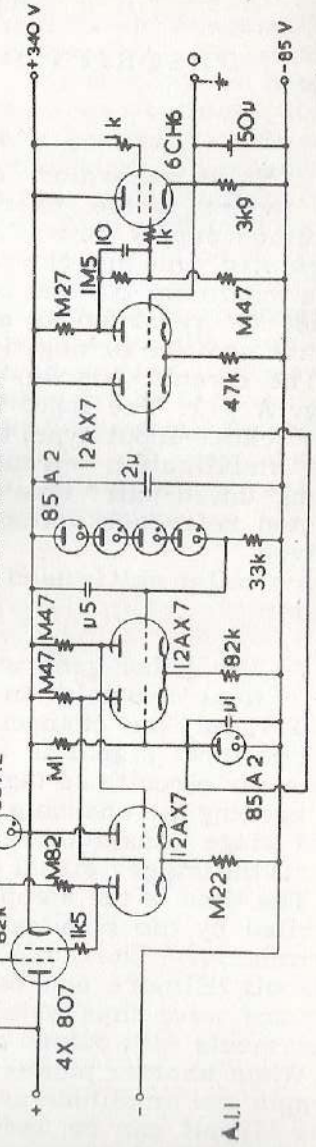


Fig. A.2.1 (above) Circuit diagram of the pulse generator.

Fig. A.1.1 (below) Circuit diagram of the electronic stabilization unit.



triodes in a Y-arrangement (Fig. A.2.2). The anode current of the lower triode (A) is allowed to flow through one of the upper triodes (B and C) which are in turn connected to each other to form a bi-stable circuit.

In the quiescent state the left triode (B) is conducting. The anode of the other one (C) is connected via a cathode follower D to a capacitor leading to the grid of triode A. The action is initiated by causing the B-C combination to switch, so that the valves A and C act like a Miller integrator (Elmore and Sands, ES-2). The charging current is determined by the setting of potentiometer II (see Fig. A.2.1). The circuit constants are chosen such that at the end of the cycle valve C starts to draw grid current. This causes the cathodes of B and C to drop in voltage, so that fly-back action is initiated. Some of the voltages occurring in the circuit are depicted in Fig. A.2.2.

The advantage of this system is that the form and size of the triggering impulse are not critical and that difficulties arising from the low mutual conductance of the suppressor grid of a pentode valve are not encountered. The output waveform is a positive pulse taken off at the anode of valve B. The top of the pulse is perfectly flat. At the end of the action there is a negative overshoot caused by grid current of valve B.

The final amplifier controls the intensity of the output pulse. The valve is normally cut off, only during the pulse the anode voltage drops over a predetermined amount (potentiometer III). All irregularities of the pulses due to the capacitive coupling to the preceding stage are smoothed out by a clamping diode.

A.3. A schematic diagram of the basic modulator circuit is shown by Fig. A.3.1. The alternating current I is divided in two parts, flowing through the valves A and B. When V = O both parts are equal and no difference voltage is developed between the terminals X and Y. When the division is such that one of the valves is conducting the greater part of I, a signal proportional to I is developed between X and Y. This voltage is also proportional to V if the mutual conductance is proportional to the grid voltage. It is thus advantageous to use valves with a nearly quadratic anode current versus grid voltage characteristic.

For suppression of the voltage V two circuits of the type indicated by Fig. A.3.1 are combined. The current

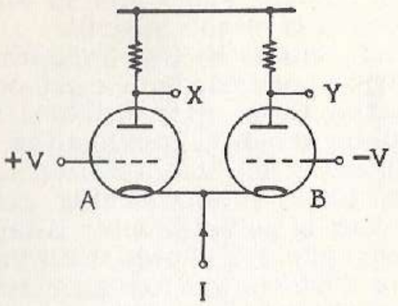


Fig. A.2.2 Phantastron circuit.

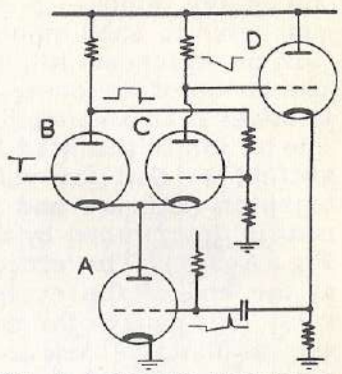


Fig. A.3.1 Basic modulator circuit.

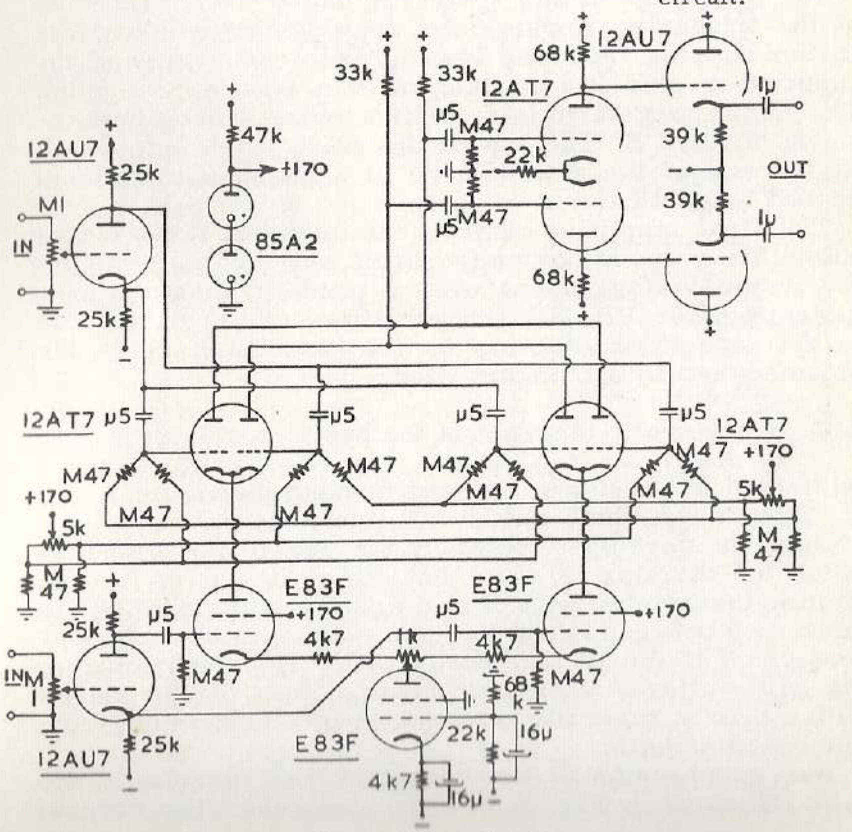


Fig. A.3.2 Modulator.

I for the second part has the opposite polarity. The complete circuit diagram of the prototype is given in Fig. A.3.2. It is seen that conventional phase inverters are used for the two input voltages and that a difference amplifier (Elmore and Sands, ES-3) and an output cathode follower handle the output voltages.

The suppression of the signal V is adjusted by proper balancing of the I-branches. The suppression of the other input signal is controlled by a potentiometer that acts like adding a d.c. voltage to the V-branch. A third potentiometer determines the mean working points of the two pairs of modulator valves with the result that distortion products associated with the V-signal are minimized. The upper input (V-signal) is normally used for the carrier signal.

A.4. The modulator circuits used in the double modulator differ only in minor aspects from the prototype described in the preceding section. The most critical valves in the low-frequency circuits and the modulator valves are heated by direct current of 150 mA drawn from the stabilized supply via two incandescent lamps of 220 V, 40 W connected in series with the series heater circuit. Some circuit elements are adapted to the higher frequencies used and in the final modulator a preamplifier is used to match the low output voltage of the band pass filter.

The oscillators are made as stable as possible without resorting to inconvenient measures such as the use of a stabilized ambient temperature. The influence of the valves on the frequency is reduced by using the valves as limiting amplifiers stabilized by d. c. negative feedback (Fig. A.4.1). The heaters of the oscillator valves are fed by the stabilized heater current of the modulator valves. Ample ventilation makes the drift of the oscillators after the warming-up period almost negligible. The oscillators are connected via cathode followers and low pass filters to the corresponding modulators.

Both signals are also fed to a mixer stage (EQ80) which produces a signal with frequency equal to the difference frequency. In order to reject high-frequency signals a third-order Butterworth filter is provided. It is of the R-C type with feedback from the cathode-follower output stage (compare Thiele, T-2). The mixer stage allows direct measurement of the net frequency shift.

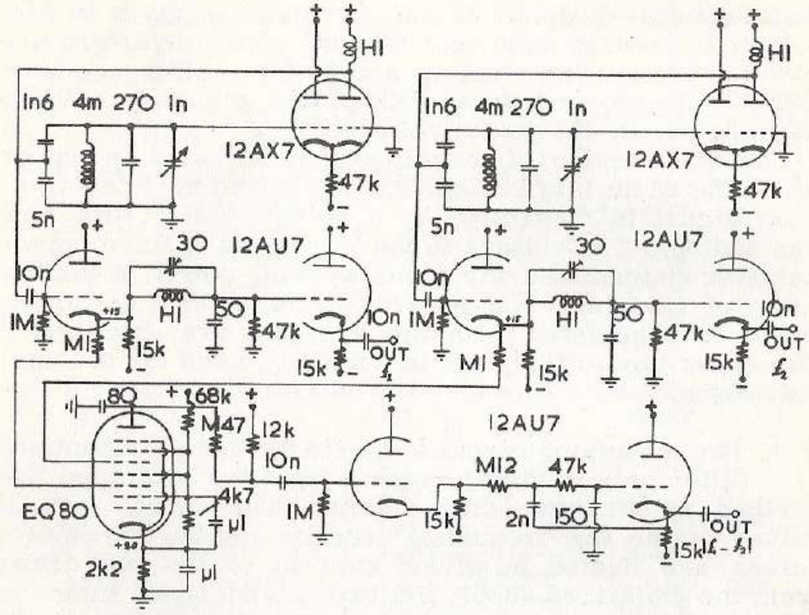


Fig. A.4.1 Circuit diagram of the oscillators and the mixer section.

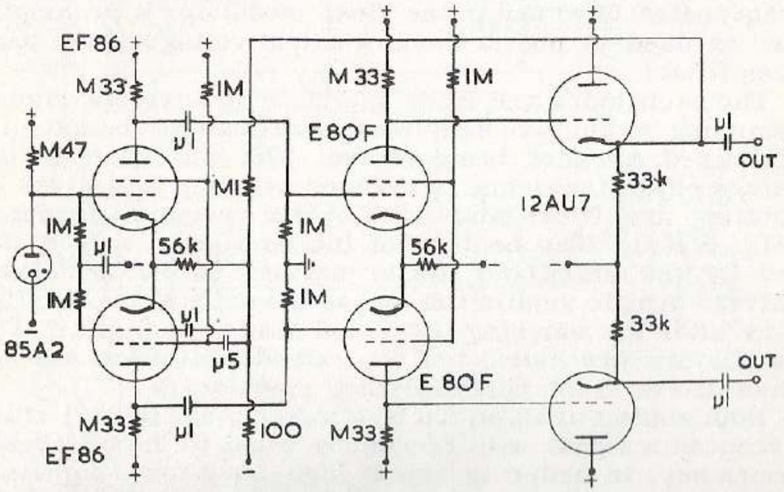


Fig. A.7.2 Circuit diagram of the noise generator.

A.5. The action of the frequency divider is described in section 5.1. Little is to be added to this on considering the circuit diagram (Fig. A.5.1).

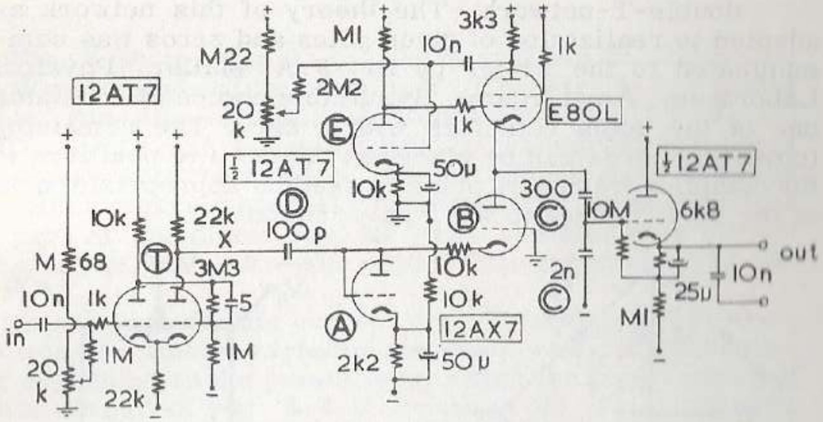


Fig. A.5.1 Frequency divider.

The principal capacitor C is divided in two parts in order to reduce the output voltage. This allows us to use a cathode follower for separating the actual circuit and the load. Capacitor C is charged by the combined action of a trigger circuit T, a capacitor D, a clamping diode A and the earthed-grid triode B. The trigger circuit converts the input signal into a square wave at the point X.

In the positive-going phase the capacitor D is charged through the diode A. In the negative-going phase the capacitor discharges through the cathode circuit of valve B. The cathode resistor limits the discharge current and prevents grid current. The charge of capacitor D is thus transmitted to capacitor C.

A configuration similar to the Puckle time-base circuit causes the capacitor C to discharge when a certain threshold voltage is passed. The threshold is determined by the working point of valve E.

The discharging action must be relatively slow in order to ensure that no remaining charge on C is left, due to the remainder of the initiating pulse. For this reason the resistor in the cathode of valve B is chosen as low as is compatible with absence of grid current. Since only small time constances are involved the adjustment needs no correction over a wide range of frequencies.

A. 6. The continuously variable filter is based upon the double-T-network. The theory of this network as adapted to realization of given poles and zeros was communicated to the writer by Dr. F. A. Muller (Physical Laboratory, Amsterdam). By suitable choice of constants one of the poles coincides with a zero. The remaining (complex) zeros can be placed at the desired positions in the complex-frequency plane by feeding appropriate parts of the input voltage to the input terminals.

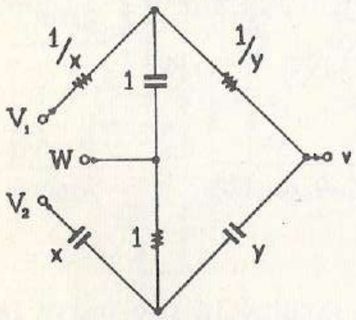


Fig. A. 6. 1 Double-T-network.

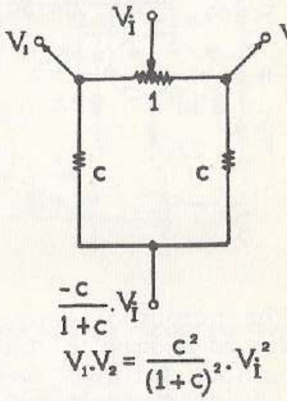


Fig. A. 6. 2. Division circuit.

The configuration is shown in Fig. A. 6. 1. The circuit constants are expressed in deliberate unities. The input terminals are connected to the voltages V_1, V_2, and W. When $x + y = 1$ the response is a rational quadratic function of the complex-frequency variable s:

$$v = \frac{V_1 + 2Ws/x + V_2 s^2}{1 + 2s/x + s^2}$$

The absolute value of the zero's is

$$\sqrt{\frac{V_1}{V_2}}$$

and the decrement

$$\pm \frac{2W/x}{\sqrt{V_1 V_2}}$$

When V_1V_2 is constant the decrement can be controlled by W and the absolute value by V_1/V_2 Fig. A. 6. 2 shows a configuration for realizing a constant product V_1V_2.

The zeros can be approximately converted into poles by insertion of the network into a feedback loop. In order to obtain separate control over zeros and poles the filter is part of the direct loop as well. The complete circuit diagram of the prototype needs little comment (Fig. A. 6. 3).

Nearly logarithmic control over W is obtained by connection of a linear variable resistor (with a fixed resistor in series) to the points from which the input voltages of the circuit of Fig. A. 6. 2 are taken off. This measure minimizes the asymmetry of the load upon the appropriate phase inverter.

The bandwidths attainable in band-pass situations were not very narrow, due to the approximations involved in the electronic design. Especially the distortion of the valves causes the circuit to have properties depending on the amplitude of the signals. In this respect the circuit is not an ideal one. For the functions here described no difficulties were encountered.

A. 7. Instead of a gas diode or a saturated vacuum diode a cold-cathode discharge valve of type 85 A 2 (Philips) is used as the noise source for the noise generator. The noise obtained in this way has a flat spectrum extending to over 16.000 c/s as is revealed by the spectrum depicted in Fig. A. 7. 1.

The noise source is followed by a two-stage differential amplifier with feedback (Fig. A. 7. 2). At the output the noise can be taken off in push-pull, a feature that is convenient in some experiments planned for the future.

A. 8. The earphone (Permoflux type PDR-10) is used to ensure that the signal is transmitted to the ear with fair fidelity. The frequency response, measured with an artificial ear (6 cc cavity), is plotted in Fig. A. 8. 1. The earphone is connected via 10 kΩ to an audio amplifier of

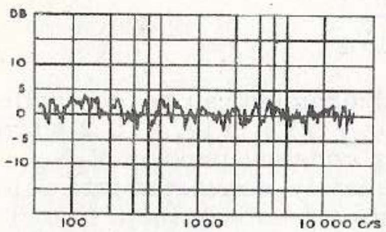


Fig. A.7.1 Noise spectrum as recorded from wave analyser.

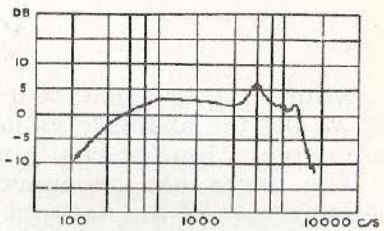


Fig. A.8.1 Frequency response of the earphone.

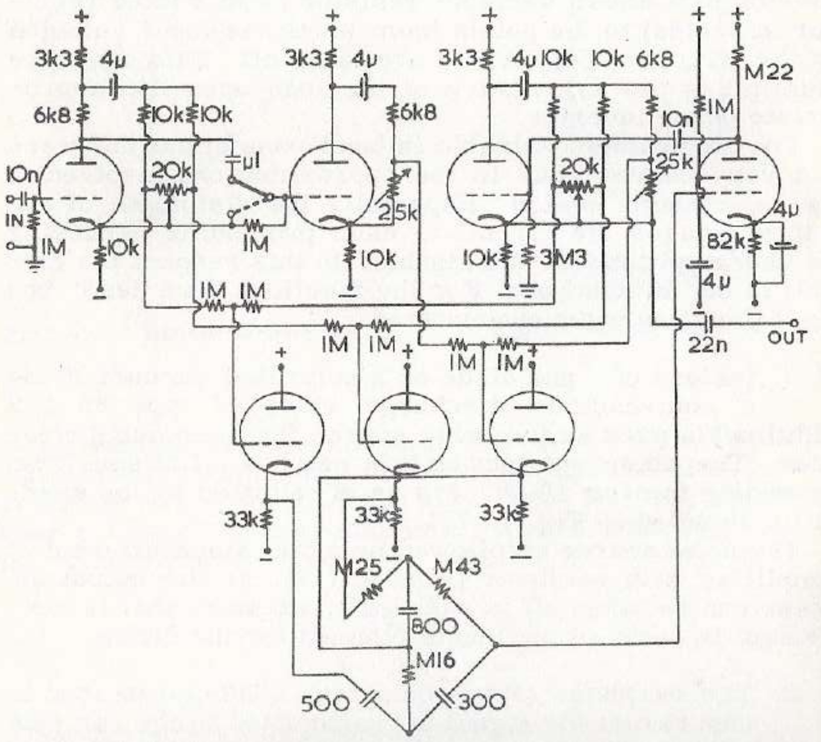


Fig. A.6.3 Circuit diagram of the continuously adjustable filter. All triodes type 12AT7, the pentode is of the type EF 86.

conventional design (provided with 20 db voltage feedback). This amplifier can, if desired, drive a loudspeaker for demonstration purposes. The normal load resistor of 8 Ω can be replaced by the 10 kΩ of the receiver circuit without causing instability due to the feedback.

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At this place I wish to thank Professor J.C.R. Licklider, who extensively discussed the phenomena with me during a visit to Amsterdam.

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I am aware that mentioning names may do injustice to others, not listed above. Therefore, I do extend my thanks to all others who contributed to this research as well.

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STELLINGEN

I

Het is onjuist alleen uit het Haas-effect en het bestaan van binaurale zwevingen te concluderen, dat de frequentie van signalen tot in hogere zenuwcentra behouden blijft.

II

De indruk, dat regressie aanwezig is alleen op grond van bepaling van aangename luidheid en pijngrens, dient, vooral in het geval van hardhorende kinderen en bij reeds lang aanwezige doofheid, met reserve te worden aanvaard.

III

Als uitbreiding van de ijking met behulp van een radioactief preparaat kunnen proportionele telbuizen, bedoeld voor kosmische-stralenmetingen, geijkt worden door het invallen van enkelvoudige deeltjes met energieën in het gebied van minimum ionisatie te registreren.

IV

De manier waarop van Dranen de rotatie-energie van CO_2 in het critische punt afleidt, is aan bedenkingen onderhevig.

J. van Dranen, J. Chem. Phys. 21 (1953), 1404.

V

De mening, als zou een isolerend membraan dat aan beide vlakken een constante lading draagt, geen kracht ondervinden ten gevolge van twee aan weerszijden geplaatste vlakke electroden met onderling gelijke potentiaal, is onjuist.

Wireless Engineer, 32 (1955), 119.

VI

Voor de realisatie van eenvoudige filters met gegeven nulpunten en polen kan men met voordeel gebruik maken van dubbele-T-netwerken in combinatie met terugkoppeling.

VII

Voor het berekenen van een laagdoorlatend filter, waarvan de responsie in een zeker frequentiegebied binnen gegeven toleranties ten opzichte van de responsie bij frequentie nul moet blijven, kan men gebruik maken van Tschebytchev-polynomen waarvan het argument een kwadratische functie van de frequentie is.

VIII

Bij de behandeling van een tegengekoppeld systeem, waarbij de tegenkoppelspanning zowel van de uitgangsspanning als van de uitgangsstroom afhangt, dient het begrip impedantie met reserve te worden gehanteerd. De beschouwingen van Clements en Childs over dynamische tegenkoppeling hebben hierom weinig betekenis.

W. Clements, Audio Eng. Aug. 1951, Mei 1952.
U. J. Childs, Audio Eng. Febr. 1952.

IX

De demping van de fundamentele resonantie van een electrodynamische luidspreker, die deel uitmaakt van een dynamische tegenkoppeling, kan onder behoud van een gunstige tegenkoppelfactor geregeld worden door in de tegenkoppelleiding een eerste-orde filter op te nemen, waarvan de pool vast ligt en het nulpunt instelbaar is.

X

De manier, waarop Olson de inwendige weerstand van de voedingsbron in het akoestisch vervangingsschema van een electrodynamische luidspreker in rekening brengt, is formeel juist maar leidt niet tot een goed inzicht in de werking.

H. F. Olson, *Elements of Acoustical Engineering*

XI

Voor practische berekeningen van akoestische systemen kan de stralingsimpedantie van een ronde zuiger in een oneindige wand vervangen worden door de impedantie van een halve bol met gelijk oppervlak in een oneindige wand. Op overeenkomstige wijze kan de impedantie van een oneindig lange strip vervangen worden door de impedantie van een halve cylinder met gelijk oppervlak per lengte-eenheid onder dezelfde omstandigheden.

XII

In tegenstelling tot de gebruikelijke uitvoering dient de zoekerlens van een twee-ogige reflexcamera uitgerust te zijn met een diafragma, dat centrale stralen tegenhoudt.

XIII

Een mogelijke verklaring van de intervallen, voorkomende in een exotische toonladder, ligt besloten in de boventonen-structuur van de gebruikte muziekinstrumenten.

XIV

Bij het beoordelen van de kwaliteit van een stemmingsmethode dient men aan de zuiverheid van de intervallen een gewicht toe te kennen, dat des te groter is, naarmate met het interval een eenvoudiger frequentieverhouding overeenkomt.

A. D. Fokker, *Recherches Musicales*, pag. 157.

XV

Bij de akademische opleiding in de natuurkunde dient aandacht geschonken te worden aan de fundamentele overeenkomst in de wiskundige behandeling van electrische, optische, akoestische, hydrodynamische en quantummechanische problemen.

XVI

Het is gewenst bij het natuurkunde-onderwijs op de VHMO-scholen uitsluitend MKS- en Giorgi-eenheden te gebruiken en niet in te gaan op de eenhedenstelsels, die op misplaatste dimensieoverwegingen gebaseerd zijn.